Additional Differential Equations Problems

1. Without solving directly, find the equilibrium values for the functions f and g given that $f(0) = 1$, $g(0) = 0$, and

$$\begin{aligned}\frac{df}{dt} &= 2g - 3f \\ \frac{dg}{dt} &= 3f - 2g\end{aligned}$$

2. Solve the above system of differential equations, following the outline below:

   (a) Show that $f + g$ must be constant.

   (b) Solve for g in terms of f.

   (c) Eliminate g from the first equation.

   (d) Solve that equation for f.

   (e) Use (b) to solve for g.

   (f) Verify that the equilibria suggested by these solutions is the same that you got in problem (1).

3. Suppose we have functions p and q with $p(0) = e$, $q(0) = 1$, satisfying the system of differential equations:

$$\begin{aligned}\frac{dp}{dt} &= p(2\ln q - 3\ln p) \\ \frac{dq}{dt} &= q(3\ln p - 2\ln q)\end{aligned}$$

   Solve this system using the outline below:

   (a) Let $f = \ln p$, and $g = \ln q$.

   (b) Compute df/dt and dg/dt, using the chain rule and the equations above.

   (c) Show that these reduce to exactly the system in problem (2).

   (d) Using the solution to (2), write explicit expressions for p and q.

(over)

4. *(Challenge Problem)* Notice that we can rewrite the system from problem (3) as

$$\frac{dp/dt}{p} = 2\ln q - 3\ln p$$
$$\frac{dq/dt}{q} = 3\ln p - 2\ln q$$

Since in this form we see that both $\ln p$ and $\ln q$ and their derivatives appear naturally in the system, we can see how we might have guessed that $f = \ln p$ and $g = \ln q$ would be a good substitution to use to try to simplify the system.

With this observation in mind, solve this system:

$$\frac{dx}{dt} = \frac{2\sin y - 3\sin x}{\cos x}$$
$$\frac{dy}{dt} = \frac{3\sin x - 2\sin y}{\cos y}$$