

EXAM 1

Math 102, Spring 2007-2008, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

1. _____

2. _____

“I have adhered to the Duke Community
Standard in completing this
examination.”

3. _____

4. _____

Signature: _____

5. _____

6. _____

7. _____

8. _____

9. _____

Total Score _____ (/100 points)

10. _____

1. (12 pts) Find the complete set of solutions to the system of equations

$$2x + 4y + 2z = 12$$

$$3x + 7y + 5z = 23$$

$$2x + 2y - 2z = 2$$

6. (12 pts) Suppose we know that the unique solution to the system

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

is $(x, y, z) = (2, 1, 0)$.

Can you ensure that the system below has at least one solution? Explain why or why not.

$$\begin{pmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 9 \end{bmatrix}$$

7. (10 pts) Let P be the unique plane that passes through the points $\vec{p} = (2, 1, 5)$, $\vec{q} = (1, 3, 5)$, $\vec{r} = (2, 2, 6)$. Find the standard equation of the plane P .

8. (12 pts) Determine if the vectors $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$ are linearly dependent or linearly independent.

9. (10 pts) Bob says that the following system of equations can be used to view T and K as endogenous variables determined by the other variables. Is he correct? Explain why or why not.

$$\begin{aligned}4T + 4M + 2K &= 2 + U \\5T + z^2M + 5K &= xy - z\end{aligned}$$

10. (12 pts) Prove that a linearly independent set of three vectors in \mathbb{R}^3 must span \mathbb{R}^3 .