Southeastern Conference for Undergraduate Women in Mathematics
Abstracts

Plenary Addresses

Suzanne Lenhart, University of Tennessee - Knoxville, *Exploring the effects of order of events in population models with discrete time*

Careful consideration of the order of events will be discussed for formulating population models with discrete time. Two examples will show how the order of events can affect optimal control results; the control actions include harvesting and augmentation of a population . One of the models is an integrodifference system which is continuous in its spatial variable.

Julianna Tymoczko, Smith College, *Generalized splines*

Splines are a well-known construction originally developed for engineering applications but now used widely in algorithms in computer graphics, differential equations, numerical analysis, and other fields. Billera and others pioneered a more theoretical approach to splines using high-powered techniques from algebra. Over the last fifteen years, geometers and topologists independently rediscovered splines, this time as objects called cohomology rings that capture many essential properties of a geometric object.
In this talk, we describe how to generalize the construction of splines to a more natural geometric and combinatorial setting, starting from a graph and a commutative ring (like the integers or the polynomials). We'll also show how powerful this construction can be: how subgraphs can simplify our calculations of splines, and how to construct a basis for different families of splines. We'll end with a number of open questions with connections to far-reaching areas of modern mathematics, like Schubert calculus, geometric representation theory, and approximation theory.
The talk will use linear algebra and arithmetic mod n, but won't require abstract algebra (or even a precise definition of a ring!).

Contributed Talks

Megan Bryant, Clemson University, *Formalizing Mathematical Developments to Support Verifying Compilers*

The development of a verifying compiler is a grand challenge in computing research. A verifying compiler uses automated mathematical and logical reasoning to guarantee the correctness of code to certain specifications. Unlike traditional testing methods, which can only demonstrate functionality in certain cases, a verifying compiler demonstrates that the code behaves correctly in every possible circumstance. The ability to verify the correctness of computer programs is a vital concern in large-scale software engineering. A successful, minimal verifying compiler needs to be supported by a flexible mathematical framework. Such a system has yet to be fully realized. We will examine the strides made in the development of this framework. Specifically, the creation of a library of mathematical theorems which supports an automated prover.

Wendy Caldwell, University of Tennessee - Knoxville, *Substance Abuse via Legally Prescribed Drugs: The Case of Vicodin in the United States*

Vicodin is the most commonly prescribed pain reliever in the United States. Research indicates that there are two million people who are currently abusing Vicodin, and the majority of those who abuse Vicodin were initially exposed to it via prescription. Our goal is to determine the most effective strategies for reducing the overall population of Vicodin abusers. More specifically, we focus on whether prevention methods aimed at educating doctors and patients on the potential for drug abuse or treatment methods implemented after a person abuses Vicodin will have a greater overall impact. We consider one linear and two non-linear compartmental models in which medical users of Vicodin can transition into the abuser compartment or leave the population by no longer taking the drug. Once Vicodin abusers, people can transition into a treatment compartment, with the possibility of leaving the population through successful completion of treatment or of relapsing and re-entering the abusive compartment. The linear model assumes no social interaction, while both non-linear models consider interaction. One considers interaction with abusers affecting the relapse rate, while the other assumes both this and an additional interaction between the number of abusers and

the number of new prescriptions. Sensitivity analyses are conducted varying the rates of success of these intervention methods measured by the parameters to determine which strategy has the greatest impact on controlling the population of Vicodin abusers. These results give insight into the most effective method of reducing the number of people who abuse Vicodin. From these models, we determine that manipulating parameters tied to prevention measures has a greater impact on reducing the population of abusers than manipulating parameters associated with treatment. We also note that increasing the rate at which abusers seek treatment affects the population of abusers more than the success rate of treatment itself.

Mikaela Cashman, Coe College, *Bar Code Localization in Images Using Neural Network and Linear Discriminant Analysis Frameworks*

We develop an algorithm for the automatic localization of 1-D bar codes in images using machine learning techniques. Despite the ubiquity of operational bar code scanners, we focus on low resolution camera-based scanners and challenging environments where traditional methods fail. We develop attributes that help distinguish bar codes from other objects in the image such as text and logos. These attributes are based on the discrete wavelet transform (DWT), the discrete Fourier transform (DFT), and gradient analysis. To create a bar code detection process robust to image distortions such as rotation, glare, noise, oblique viewing angle, uneven and dim illumination, and an abundance of surrounding text, we use the information from our individual detection methods in neural network, linear discriminant analysis (LDA) frameworks, and simple boosting. We analyze the effectiveness of these attributes and report on performance for a range of degraded images.

Kelly Cercy, Furman University, *Math in the City: Parking Optimization in Downtown Greenville*

The city of Greenville, located in upstate South Carolina roughly between Atlanta and Charlotte, owns and operates nine garages that offer monthly parking rates. City representatives from Economic Development say the city is growing, and employees of the Department of Public Works' Parking Services see the parking picture as one that is filling up quickly. Some garages are already reaching capacity, and there will be added pressure as the economy

grows. Thus the city asked Furman University to research downtown parking for the more than 4,000 monthly pass holders (primarily from businesses downtown). We set out to answer these questions through integer programming with the goal to make the commute for pass holders as convenient as possible. Convenience translated to minimizing total walking distance, a perspective that was new to the city. We will study current usage and industry standards to improve logistics, and then analyze efficiency and capacity to determine the need and location for a new garage.

Carmen Cox, Duke University, *How Philosophy can add a New Dimension to Math*

Many characterize math based on its extreme ability to be universal, logical, and straightforward. Over time, whether it is recognized socially or not, people have relied on, and perhaps even found comfort, in mathematics; it is unique to find something in this world that can separate itself, at least to some degree, from uncertainty and flaws. However, it is our opinions of math, the perspectives, the fondness, the questioning and dedication that color in the shades and shadows that make math something that is truly beautiful. From Pythagoras opinion on the divinity of math to Bertrand Russells paradox, philosophy has helped add color and growth to a subject area that some may forget is ultimately human.

Elise Darlington, Morningside College, *Modeling Exciton Dynamics with Markov Chains*

In organic photovoltaics, studying the properties of exciton movement is essential for determining the major causes of device inefficiency. In this work, a Markov chain method is developed to capture the dynamic behavior of excitons in a disordered energetic landscape. It was found the Markov Chains are far more efficient, compared to Monte Carlo simulations, to quantify some figures of merit related to exciton diffusion such as diffusion length, transport time and exciton dissociation efficiency. This study was primarily focused on studying the mean diffusivity for a device as energetic disorder varies. In particular, it was determined that exciton diffusivity is not only lower with increases in energetic disorder, but is decreasing over the life of an exciton. Moreover, evidence was presented that the probability distribution for the

location of an exciton at a given time is normally distributed and has a radially symmetric distribution function.

Melissa Gaddy, Wofford College, *Restoration and Analysis of Apollo Lunar Data*

The Lunar Ejecta and Meteorites (LEAM) Experiment on Apollo 17 was designed to measure hypervelocity particles that collide with the surface of the Moon. Original data analyses from the LEAM experiment yielded unusually high numbers of low-velocity impacts during the passage of the terminator. Recently, a question about electrical noise on the LEAM instrument has arisen, and there is some speculation that noise generated by the heaters and other instrumentation may have influenced the results. The current analysis examines the data to see if there are correlations between the unexpected number of events and the times that the power levels were fluctuating. Analyses have revealed unusual patterns in the accumulators of the UP, EAST, and WEST sensor. Further examination of this outcome may reveal electrical interference, and additional channels of housekeeping data can be studied.

Renata Gerecke, Pomona College, *Factor Groups of Knot and LOT Groups*

It is difficult to determine whether, given a finite, balanced group presentation, the group is finite or infinite. We study this problem in the context of knot groups and label orientated tree (LOT) groups. More specifically, we are looking at factor groups of knot & LOT groups by powers of meridians. This is in the spirit of Coxeter's work on the factor groups of braid groups. Indeed, our findings generalize Coxeter's work from the three-strand braid groups to knot groups.

Desiree Ippolito, Florida Southern College, *Scheduling Office Hours: A Timetabling Problem*

At Florida Southern College teachers are required to have 5 office hours a week to help students. Many teachers find that far fewer students are coming to office hours than should be, and the most common excuse is they are unavailable during the scheduled office hours. Unfortunately, faculty had no way of scheduling office hours based on student's schedules, until now! Using

various timetabling techniques we created a program that takes in student data and teacher preferences and returns the best office hours.

Lynnique Johnson, Winthrop University, *Using Linear Cancer Networks to Model Photodynamic Therapy*

Photodynamic therapy (PDT) is a treatment used in the early stages of certain cancers which involves a localized injection of a photosensitizer followed by exposure to a light source that releases a cytotoxic agent, singlet oxygen, into surrounding cells. We create a set of eight ordinary differential equations that model photodynamic therapy (PDT) within a linear cancer network. The cancer network consists of cancer stem cells which can either differentiate into tumor cells or create more stem cells. The model reveals three possible behaviors of the cancer: a stable equilibrium state of coexisting cancerous and healthy cells, an extended remission with eventual recurrence, or permanent remission. Stability and sensitivity analyses are performed to support numerical simulations.

Rachael Keller, Louisiana State University, *Finding Hadamard Difference Sets*

Some sets lying in groups can, without the identity element of the group, form entire copies of the group when combined with the inverses of the elements in the set. Difference sets are these such sets; Hadamard Difference Sets are sets composed of $2m^2 - m$ elements that, when each element is combined with the inverse, or "difference," of every other element of the set, together form the entire group $m^2 - m$ times.
In this talk, we will introduce Hadamard Difference Sets and explain the methods for finding them. Our implementation of them on a large-scale, observations of the characteristics of sets for particular groups, and ideas for new construction methods will be discussed.
This is joint work with Alec Biehl, Kevin Halasz, Maryna Longnickel Dylan Peifer,Ken Smith, and Jason Steinberg.

Olivia Manley, Winthrop University, *A mathematical model of cancer networks with radiation therapy*

Rather than the traditional explanation of cancer as the result of mutated genes that cause uncontrolled cell growth, a theory of mutated developmental control networks has been developed by Oxford scientist Eric Werner. This research examines one such developmental control network, a linear cancer network. A linear cancer network describes a system where cancerous growth is driven by cancer stem cells, which produce more cancer stem cells and other nondividing tumor cells. A mathematical model that depicts the behavior described by this new paradigm of cancer growth is proposed. Then, treatment in the form of radiation therapy is introduced, and the resulting effects on each cell population are explored. This research aids in the understanding of cancer, its growth, and how treatment may interact with it. The proposed mathematical model uses a nonlinear system of three ordinary differential equations that describe the growth of cancer stem cells, tumor cells, and healthy cells. Equilibrium points are analyzed and interpreted to uncover the long-term behavior of the model. Numerical simulations of the system confirm the stability analysis and reveal behavior such as failed treatment, cure states, and tumor recurrence for varying levels of radiation effectiveness.

Maria Meyer, University of Kentucky, *Nim on Groups*

The traditional game of Nim comprises of two players who take turns removing objects from distinct piles. The player who takes the last object is the winner. We consider the game Nim on Cayley graphs of finite groups, where the piles are located on the vertices and the number of objects in each pile is denoted as the *weight* of the vertex. In this version of the game, a player wins by trapping the opponent on a vertex with weight zero so he or she is unable to further reduce the weight of that vertex. We examine winning strategies for Nim on Cayley graphs of cyclic groups, dihedral groups, and the Quaternions, among others.

Faith Miller, Appalachian State University, *On the Crossing Number of the* $K3,3,3$ *Tripartite Graph*

Graph crossing numbers are considered NP-Hard, with no easy theorems set up to solve them. We will be discussing a brief history of crossing numbers. Specifically, by using D.J. Kleitman and Kouhei Asano's paper, we can minimize the upper and maximize the lower bounds of the $K3,3,3$ tripartite

graph. We will discuss techniques on examining the bounds for this graph.

Laura Ohm, University of Washington, *Folic acid supplementation and recovery from spinal cord injury: a mathematical model*

The folate cycle is vital to the human body, serving, among other functions, as an important step in DNA methylation. Injuries to the spinal cord and central nervous system (CNS) tissues have been shown to alter folate distribution in the body, causing a decrease in concentration of methionine synthase, the main methyl donor. Folic acid, a folate form rarely found in natural foods, has been shown to increase neuronal regeneration following CNS injury when taken as a dietary supplement. Additionally, experimental data shows biphasic concentration changes in methionine synthase and other key players in the folate cycle in response to increasing doses of folic acid, suggesting that moderate supplementation best enhances regeneration. To determine a possible mechanism for the observed biphasic neuronal regeneration, an existing ODE model of folate metabolism was modified to account for CNS injury and subsequent folic acid supplementation. The resulting model suggests that the experimentally observed changes to the folate cycle may relate to the cells folic acid uptake scheme, especially concentration-dependent changes in folic acid receptor FolR1 activity and competitive inhibition of folic acid uptake by other folate substrates.

Emili Price, Winthrop University, *Combinatorics of Quartet Amalgamation*

One technique for reconstructing phylogenetic trees is inputting a set of quartet trees (containing four taxonomic units) and amalgamating them into a single supertree. We prove that the minimal number $k(|X|)$ such that every compatible quartet system Q, with $Q \geq k$ defines a unique tree is $\binom{|X|}{4} - (|X| - 4)$. Moreover, we prove that this bound is optimal because we can construct quartet systems $\binom{|X|}{4} - (|X| - 3)$ that do not define a unique tree. We examine quartet subsystems to identify necessary and sufficient conditions required to ensure that the subsystems contain a sparse set of quartets meeting the criteria of Bocker for defining a unique supertree. Moreover, we prove a lower bound, $p(|X|, k)$ for the percent of time a unique tree will be reconstructed for any number of taxa and and an arbitrary set of quartets. Our technique can be applied to quartet tree amalgamation algorithms to

increase the speed at which they construct supertrees.

Amber Robertson, University of Southern Mississippi, *Chebyshev polynomial approximation to solutions of ordinary differential equations*

In this thesis, we develop a method for finding approximate particular solutions for second order ordinary differential equations. We use Chebyshev polynomials to approximate the source function and the particular solution of an ordinary differential equation. The derivatives of each Chebyshev polynomial will be represented by linear combinations of Chebyshev polynomials, and hence the derivatives will be reduced and differential equations will become algebraic equations. Another advantage of the method is that it does not need the expansion of Chebyshev polynomials. This method is also compared with an alternative approach for particular solutions. Examples including approximation, particular solution, a class of variable coefficient equation, and initial value problem are given to demonstrate the use and effectiveness of these methods.

Shan Shan, Agnes Scott College, *Periodicity of Third-Order Linear Recursive Sequences*

In 2011, Franzel, Psalmond, and Tobiasz provided a necessary and sufficient condition for a number k to be the period length of an integer sequence described by a second-order recurrence relation modulo a prime p. In this talk, we extend their techniques to sequences described by third-order recurrence relations. We show that any such sequence modulo p has period length dividing $p^3 - 1$, $p^2 - 1$ or $p(p - 1)$. Conversely, any divisor of these three numbers can be realized as a period length for some such sequence.

Anne Talkington, Duke University, *A Mechanistic Model for Tumor Growth and Response to Cancer Treatment*

Mathematical models can provide mechanistic insight into tumor growth, and consequently enable more effective cancer treatment. Current knowledge about tumor growth has traditionally been derived from qualitative model analysis. Tumor growth patterns are generally exponential at the onset; however, they begin to level off as oxygen is unable to efficiently reach central

cancer cells. The type of cancer and patients medical history influence the degree to which this slowing of tumor growth occurs. Upon treatment, the tumors growth rate will generally slow further, or the tumor will decrease in size. Post-treatment tumors appear to recover and grow exponentially, but have thus far only been studied with statistical regression. Logistic and Gompertzian models have previously been used to describe the shape of a cancer growth curve. My objective was to find a more analytical, mechanistic model for tumor growth and response to treatment. I have investigated a modified exponential growth pattern and the Modified Alternating Maclaurin Series (MAMS), previously applied to microbial populations, as potential models to describe tumor growth and its limitations (including the treatment agent, space, nutrients, and hypoxia). I have compared cancer growth models and fit them against experimental data. The success of certain targeted therapies relies on the growth phase of the cancer. Understanding mechanistic growth model parameters can aid in assessing effectiveness in xenograft drug trials. Therefore, greater knowledge and understanding of pre- and post- treatment tumor growth patterns can result in more efficient administration of initial treatments, and successive treatments in the case of relapse.

Stella Watson, Furman University, *A Version of the Firefighter Problem in which the Fighter Walks a Continuous Path on a Square Grid*

This presentation examines a version of the well-studied firefighter problem. In this version of the problem, a fire spreads along adjacent nodes on a square graph, and the fighter attempts to contain the fire by protecting nodes in a continuous path around the fire. He is able to completely surround the fire from any starting position when he protects at least 4 nodes every time step. When protecting 3 nodes per time step, he can only completely surround the fire from certain starting positions, and when protecting 2 nodes per time step he cannot completely surround the fire from any starting position. Under certain conditions, the fighter's path is governed by the golden ratio and the Fibonacci numbers.

Corey Yeates, University of Southern Mississippi, *Application of Linear Sequences to Cryptography*

Cryptography is a century–old technique of secretly transferring information

between parties. Linear recurrences are the chosen method of encryption and decryption here. The Fibonacci sequence allows for the flexibility of encoding any number desired based on a particular encoding technique used in the film *Sherlock Holmes: A Game of Shadows*. The main goal is to find other linear recurrences that possess characteristics similar to the Fibonacci sequence to use as suitable substitutes for encoding.
Different sequences were analyzed based on a number of criteria. In order for a sequence to be a candidate, it had to be first deemed a possible sequence based on the nature of its roots. Once it passed this test, a particular method was developed for showing that a sequence could definitely be used to encode a set of numbers, and then applied to various sequences.

Kinsey Zarske, University of Southern Mississippi, *Surfaces of Revolution with Constant Mean Curvature in Hyperbolic 3-Space*

In this talk, we discuss how to obtain the differential equation of the profile curve for a surface of revolution with constant mean curvature $H = c$ in hyperbolic 3-space $\mathbb{H}^3(-c^2)$ of constant curvature $-c^2$. By solving the differential equation, we are able to construct surfaces of revolution with constant mean curvature $H = c$ in $\mathbb{H}^3(-c^2)$. Intriguingly, as $\mathbb{H}^3(-c^2)$ flattens out to $\mathbb{E}^3$, Euclidean 3-space with $c \to 0$, the limit of the surfaces of revolution with $H = c$ in $\mathbb{H}^3(-c^2)$ becomes a catenoid, the minimal surface of revolution in $\mathbb{E}^3$.

Meina Zhou, Agnes Scott College, *On the longest period of a linear recursive sequence modulo a prime*

We investigate on the period length of sequences described by a second order linear recurrence relations a_0=0, a_1=1, a_n= $c_1 a_{(n-1)}$+ $c_2 a_{(n-2)}$ We prove that there are $\phi(p^2-1)/2$ ways to choose c_1 and c_2 to get a sequence of longest order. Additionally, we show that the number of different choices of c_1 is $\phi(p-1)$; and for a given c_1, the number of choices of c_2 is $\phi(p^2-1)/(2\phi(p-1))$.
