

EXAM 2

Math 102, Spring 2006-2007, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

1. _____ (/20 points)

2. _____ (/20 points)

3. _____ (/20 points)

4. _____ (/20 points)

5. _____ (/20 points)

“I have adhered to the Duke Community
Standard in completing this
examination.”

Signature: _____

Total _____ (/100 points)

1. (a) Let $f(x, y, z) = \begin{bmatrix} \ln\left(\frac{x^2+y^2+z^2}{3}\right) \\ 2xy^2z - x^2y \end{bmatrix}$. Use the total derivative of f to estimate the value $f(1.01, .97, .99)$.

- (b) The function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is given by $f(\vec{x}) = A\vec{x}$, with

$$A = \begin{pmatrix} 2 & 3 \\ 7 & 1 \\ 5 & 0 \end{pmatrix}$$

Compute the derivative matrix Df .

2. (a) Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, with $f(3, 1, 2) = (3, 1, 2)$, and

$$Df(3, 1, 2) = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 3 & 2 \\ 1 & 5 & 4 \end{pmatrix}$$

If $g = f \circ f$, and $g(x, y, z) = (p, q, r)$, compute $\frac{\partial r}{\partial y}(3, 1, 2)$. (*Make sure to explain how the value of $f(3, 1, 2)$ is used in this computation!*).

- (b) The production level P of a company is known to depend on two factors, A and B , as given by $P = 5A^{1/2}B^{1/2}$. At a time when $A = 16$ and $B = 4$ we know that $\frac{dP}{dt} = 10$ and $\frac{dA}{dt} = 2$. What is $\frac{dB}{dt}$?

3. (a) What is the rate of change of the function $f(x, y, z) = x^2 - y^2z - xz^2$ at the point $\vec{a} = (1, 1, 1)$ when moving at unit speed in the direction of the vector $(6, 2, 3)$?

- (b) In what (unit vector) direction from the point $\vec{a} = (1, 1, 1)$ is the function $f(x, y, z) = x^2 - y^2z - xz^2$ decreasing the most quickly?

4. Suppose we know that $x^2yz + xy^2 - z^2 + 1 = 0$. At the point $(0, 2, 1)$ on this surface, which of the variables can be expressed in terms of the other two?

5. Consider the system of equations

$$\begin{aligned}wxy - xyz + 3wz &= 4 \\w^2 + z^2 + xy &= 7\end{aligned}$$

near the point $(w, x, y, z) = (1, 2, 1, 2)$.

Show that we can locally view y and z as functions of w and x , and compute $\frac{\partial z}{\partial w}$.