

EXAM 3

Math 102, Spring 2006-2007, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

1. _____ (/20 points)

2. _____ (/10 points)

3. _____ (/30 points)

4. _____ (/20 points)

5. _____ (/20 points)

“I have adhered to the Duke Community
Standard in completing this
examination.”

Signature: _____

Total _____ (/100 points)

1. (a) Determine if the matrix A is positive or negative definite or semidefinite, or indefinite.

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -5 \end{pmatrix}$$

- (b) Determine if the matrix A above is positive or negative definite or semidefinite, or indefinite, subject to the constraint that $x - 5z = 0$.

2. Find all of the critical points of the function

$$f(x, y) = x^3 - 3xy^2 + 6y^2$$

3. Find the absolute maximum value of the function $f(x, y, z) = x + y + z$ subject to the constraints $z - x - y - 4 \leq 0$ and $x^2 + y^2 - z \leq 0$.

(extra room if needed)

4. The price of real estate in a particular rural area is known to be proportional to the distance it is away from a smelly landfill, located at the origin (location, location, location!). Specifically, the price per square kilometer is $P(r) = (\$250,000)(r)$, at a distance r kilometers away from the origin.

A developer would like to purchase a circular plot of land of radius 1km, centered 1km north of the landfill. How much should he expect to have to pay for this purchase?

5. Compute the integral of the function $f(x, y, z) = x + yz$ over the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 3$, $z = 2 - y$.