

EXAM 2

Math 102, Fall 2007-2008, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

1. _____ (/20 points)

2. _____ (/20 points)

3. _____ (/20 points)

4. _____ (/20 points)

5. _____ (/20 points)

“I have adhered to the Duke Community
Standard in completing this
examination.”

Signature: _____

Total _____ (/100 points)

1. (a) Let $f(w, x, y, z) = (\tan(wx - yz), wxe^{yz})$. Use the total derivative of f to estimate the value $f(.99, 2.03, 1.98, 1.01)$.

- (b) Find the equation of the tangent plane to the surface described by the equation

$$x^2e^y + xz^2 = 6$$

at the point $(2, 0, 1)$.

2. Suppose we have $x = e^{u^2v^2-v}$, $y = e^{u^2-v^2}$, $p = e^{x^4-y^2}$, $q = e^{x^2-y^3}$.

(a) Compute the derivative matrix for the function $g(u, v) = (x, y)$.

(b) Compute the derivative matrix for the function $f(u, v) = (p, q)$ when $(u, v) = (1, 1)$.

3. (a) Use the gradient vector to compute the directional derivative of the function $f(x, y) = x^2 + xy$ at the point $(1, 2)$, in the unit vector direction of the vector $(5, 12)$.

(b) Compute the above directional derivative directly from the definition of the directional derivative.

4. Consider the system of equations

$$\begin{aligned}v^2x + wxy - xz &= -1 \\vw + vx + wx + xz &= 1\end{aligned}$$

near the point $(v^*, w^*, x^*, y^*, z^*) = (0, 0, 1, 1, 1)$.

(a) Show that we can locally view the variables v and w as functions of the variables x , y , and z .

(b) Viewing the variables as above, compute $\frac{\partial w}{\partial x}$.

5. (a) Given the matrix A below, determine if the quadratic form $Q(\vec{x}) = \vec{x} \cdot A\vec{x}$ is positive or negative definite or semidefinite, or indefinite.

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{pmatrix}$$

(b) Find all of the critical points of the function $f(x, y) = x^2 - xy^2 + 3y + y^4$.