

# EXAM 3

Math 102, Fall 2007-2008, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING  
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name \_\_\_\_\_

ID number \_\_\_\_\_

“I have adhered to the Duke Community  
Standard in completing this  
examination.”

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

Signature: \_\_\_\_\_

Total Score \_\_\_\_\_ (/100 points)

1. (20 pts) Find the shortest distance between the origin and a point on the surface defined by  $z = e^{-(x^2+y^2)}$ .

2. (20 pts) Find the absolute maximum value of the function  $f(x, y) = xy$  subject to the constraints  $x \geq 0$ ,  $y \geq 0$ ,  $x \leq 2$ ,  $y \leq 2$ ,  $x + y = 3$ .

3. (20 pts) Compute the integral of the function  $f(x, y) = x - y^2$  over the domain that is bounded by the curves  $x = y^2$  and  $y = x - 2$ .

4. (20 pts) The “probability density function” for the governmentally determined economic parameters  $x$ ,  $y$ , and  $z$  is given by  $f(x, y, z) = 4xy$ . A stock speculator believes that the price of a particular stock will go up if those parameters fall in the domain  $D$  defined by  $x \geq 0$ ,  $y \geq 0$ ,  $x^2 + y^2 \leq 1$ , and  $0 \leq z \leq 1$ .

The probability of this happening is given by the integral

$$\iiint_D f(x, y, z) dV$$

Compute this probability.

5. (20 pts) The population of a circular island (of radius 3 miles) is concentrated at the center of the island, with the population density (in thousands of people per square mile) given by  $\delta(x, y) = 4e^{-r^2}$ , where  $r^2 = x^2 + y^2$ . Find the total population of the island.