EXAM 3

Math 102, Spring 2007-2008, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ______________________________

ID number____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
8. __________

Total Score __________ (/100 points)

1. *(14 pts)* Consider the system of equations

$$\begin{aligned} e^v + 3w + x^2 - y^3 + z^4 &= 2 \\ 3v + e^w + x^4 + y^2 - z^3 &= 2 \end{aligned}$$

near the point $(v^*, w^*, x^*, y^*, z^*) = (0, 0, 1, 1, 1)$. Show that the variables v and x can be viewed as implicit functions of the other three variables, and then compute the partial derivative $\frac{\partial x}{\partial z}$.

2. *(12 pts)* Find all of the critical points of the function $f(x, y) = x^3 - 12xy^4 - 48y^4$.

3. *(12 pts)* Determine if the matrix below represents a quadratic form that is positive definite, positive semidefinite, negative definite, negative semidefinite, or indefinite.

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 2 & 1 \\ 0 & 3 & 8 \end{pmatrix}$$

4. *(14 pts)* Find the absolute maximum value of the function $f(x, y, z) = x^2 - y^2$, subject to the constraints that $x + y \geq 0$, $x - y \geq 0$, $z \geq 0$, and $x + z = 1$.

5. *(12 pts)* Write down (but do not evaluate!) the explicit Riemann sum, in terms only of the dummy indices i, j, and n, that is the definition of the integral $\iint_R (x^3 + xy)\,dx\,dy$, where R is the rectangle $[0,3] \times [2,6]$.

6. *(12 pts)* Compute the value of the integral $\iint_D 4x^3\,dA$, where D is the region inside the ellipse $x^2 + 4y^2 = 16$ and above the line $y = -1$, with $x \geq 0$.

7. *(12 pts)* Set up, but do not evaluate, a triple nested integral that represents the mass of the region bounded by the planes $y = 0$, $y = x$, $z = 0$, $x + y + z = 2$, where the density is given by $\delta = e^{xyz}$.

8. *(12 pts)* Use a double integral to compute the area that is inside of the circle $(x-1)^2+y^2 = 1$ and outside of the circle $x^2 + y^2 = 1$.