

EXAM 2

Math 102, Spring 2008-2009, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

“I have adhered to the Duke Community
Standard in completing this
examination.”

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Signature: _____

Total Score _____ (/100 points)

1. (14 pts) Determine if the set of vectors below forms a basis for \mathbb{R}^3 :

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2. (14 pts) In what unit vector direction is the function $f(x, y, z) = x^2y - z^3 + 4xz$ increasing most quickly from the point $(1, -1, 2)$?

3. (24 pts) In this problem we consider the curve C that is the unique line passing through the points $(2, 1)$ and $(4, 5)$.
- (a) Find a function $f : \mathbb{R}^a \rightarrow \mathbb{R}^b$ such that C is the graph of f , and make sure to identify explicitly the values of a and b .
- (b) Find a function $g : \mathbb{R}^c \rightarrow \mathbb{R}^d$ such that C is a level set of g , and make sure to identify explicitly the values of c and d .
- (c) Find a function $h : \mathbb{R}^i \rightarrow \mathbb{R}^j$ that represents C parametrically, and make sure to identify explicitly the values of i and j .

4. (14 pts) At the company where you work, the expected profit (P) from production with input parameters r , s , and t is given by

$$P = \left(e^{6-r^2-s^2-t^2} \right) (r^2 s)$$

At the moment the company has the parameters set at $(r^*, s^*, t^*) = (2, 1, 1)$. You have heard through the grapevine that the company will be changing s and t by amounts $ds = .01$ and $dt = .02$, and also that the company expects to increase its profits by $dP = .24$. What can you infer about planned changes to r ?

5. (14 pts) Consider the functions below:

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 - x_2 \\ x_2^2 - x_1 \end{bmatrix} \quad \text{and} \quad g\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_1 x_2 \end{bmatrix}$$

and the composition defined by

$$h(\vec{x}) = f(g(\vec{x}))$$

Use the chain rule to find the derivative matrix Dh at the point $(2, 1)$.

6. (20 pts) Consider the system

$$\begin{aligned}u^2 - uv^2 + vx - uy^2 + yz &= 1 \\u + v - xy + 2yz &= 8\end{aligned}$$

near the point $(u^*, v^*, x^*, y^*, z^*) = (1, 2, 1, 1, 3)$. Determine if the variables x and y can be viewed locally as implicit functions of the other variables. If they can, compute their partial derivatives with respect to u at the given point.