

# EXAM 3

Math 102, Spring 2008-2009, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING  
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name \_\_\_\_\_

ID number \_\_\_\_\_

1. \_\_\_\_\_

2. \_\_\_\_\_

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7. \_\_\_\_\_

“I have adhered to the Duke Community  
Standard in completing this  
examination.”

Signature: \_\_\_\_\_

Total Score \_\_\_\_\_ (/100 points)

1. (10 pts) Consider  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  given by  $f(x, y) = (e^x \cos y, e^x \sin y)$ . Is  $f$  locally invertible at every point in its domain? If not, find all of the counterexamples.

2. (10 pts) Determine if the following matrix is positive (or negative) definite (or semidefinite), or indefinite:

$$\begin{pmatrix} 5 & 0 & 7 & 0 \\ 0 & 0 & 3 & 2 \\ 7 & 3 & 0 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}$$

3. (16 pts) Find the absolute maximum value of the function  $f(x, y) = 32x - x^4 + 108y - y^4$  on  $\mathbb{R}^2$ .

4. (16 pts) Find the absolute maximum value of the function  $f(x, y) = x^3 + 3y^3 - 12x + 7y$  on the solid rectangle  $R$  in the  $xy$ -plane with corners at  $(0, 0)$ ,  $(3, 0)$ ,  $(0, 2)$ ,  $(3, 2)$ .

5. (16 pts) Find the absolute maximum value of the function  $f(x, y, z) = z$  with constraints  $h_1 = z - x^3 + 6y^2 = 0$  and  $h_2 = x^2 + y^2 = 25$ .

6. (16 pts) Square County is in the shape of a square, with side length of 10 miles and the origin at the southwest corner of the county. The Sheriff of Square County has data that suggest that the density (in thousands of people per square mile) who support his candidacy for County Commissioner is given by  $f(x, y) = xye^{-x^2}$ . How many supporters can the Sheriff reasonably claim to have? (Feel free to use the approximation  $e^{-100} \approx 0$ .)

7. (16 pts) The region  $R$  is the region inside the unit sphere and also inside the paraboloid with equation  $3z = 2x^2 + 2y^2$ . Set up, but do not evaluate, a triple integral representing the mass of  $R$  where the density is given by  $\delta(x, y, z) = e^{xyz}$ .