EXAM 2

Math 102, Fall 2009-2010, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number_____

1. _____
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"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

Total Score _____ (/100 points)

1. *(15 pts)* Use the total derivative to estimate the value of $f(1.01, 0.02, 2.98)$, where the function f is defined by $f(x, y, z) = x^2 z e^{xyz} - x^2 yz$.

2. *(10 pts)* Suppose we have $x = uv^2$, $y = u^2 - v^3$, $z = 3uv$, $r = xyz^2 - z^2$, $s = x^3 - y^3 - z^3$. Use the chain rule to compute the value of $\frac{\partial r}{\partial v}$ when $u = 1$ and $v = 2$.

3. *(15 pts)* At the point $\vec{x}^* = (x^*, y^*, z^*)$, the differentiable function $g : \mathbb{R}^3 \to \mathbb{R}^1$ is increasing most quickly in the direction parallel to the vector $(-2, 6, -3)$, and the directional derivative in that direction is 35. What is the directional derivative of g at this point in the direction that is parallel to the vector $(12, 3, -4)$?

4. *(15 pts)* The variables w, x, y, and z are related by the equation $x^4y^2 - 2wxz + w^3yz = -2$. Compute $\frac{\partial w}{\partial x}$ at the point $(w^*, x^*, y^*, z^*) = (1, 2, 2, 3)$.

5. *(15 pts)* The variables p, q, r, s, and t are related by the system

$$\begin{aligned} pq^2 - qr^2 - rs^2 + st^2 &= 4 \\ p^2q + q^2r + r^2s - s^2t &= 2 \end{aligned}$$

Show that near the point $(p^*, q^*, r^*, s^*, t^*) = (1, 2, 0, 2, 0)$ we can view q and r as implicitly defined functions of the other variables, and compute $\frac{\partial r}{\partial p}$ at this point.

6. *(15 pts)* Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (y, x^3 + xy)$. A what points in the domain is this function locally invertible?

7. *(15 pts)* Let Q be the quadratic form defined by the matrix

$$A = \begin{pmatrix} -3 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Determine if this quadratic form is positive (or negative) definite (or semidefinite), or indefinite.