EXAM 3

Math 102, Fall 2009-2010, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ____________________________________

ID number________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________

Total Score ____________ (/100 points)

1. *(15 pts)* Find all of the critical points of the function $f(x, y) = \frac{1}{3}x^3 - 4xy^2 + y^2$.

2. *(15 pts)* Find the absolute maximum value of the function $g(x, y) = 2xy - 16x^2 - 4y^2$ inside the interior of the regular octogon centered at the origin with vertices at the points $(\pm 1, 0)$, $(0, \pm 1)$, $(\pm\sqrt{\frac{1}{2}}, \pm\sqrt{\frac{1}{2}})$.

3. *(15 pts)* Find the absolute maximum value of the function $f(x, y) = x^2y^3$ on the solid triangle with vertices at $(0, 0)$, $(2, 0)$, $(0, 3)$.

4. *(20 pts)* Suppose we are trying to find the maximum value of the function $f : \mathbb{R}^4 \to \mathbb{R}$, subject to the three constraints $h_1 = 0$, $h_2 = 0$, $h_3 = 0$. All of these functions are known to be smooth. We also know the following about the points $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ on the constraint set:

$$\nabla f(\vec{a}_1) = (3,2,5,2) \quad \nabla h_1(\vec{a}_1) = (1,3,2,0) \quad \nabla h_2(\vec{a}_1) = (-2,1,4,0) \quad \nabla h_3(\vec{a}_1) = (4,0,1,0)$$

$$\nabla f(\vec{a}_2) = (3,1,1,2) \quad \nabla h_1(\vec{a}_2) = (1,0,0,2) \quad \nabla h_2(\vec{a}_2) = (0,1,0,0) \quad \nabla h_3(\vec{a}_2) = (2,0,1,0)$$

$$\nabla f(\vec{a}_3) = (0,2,5,6) \quad \nabla h_1(\vec{a}_3) = (1,2,0,0) \quad \nabla h_2(\vec{a}_3) = (5,1,0,0) \quad \nabla h_3(\vec{a}_3) = (3,4,0,0)$$

Which of these points are critical points? (Make sure to justify your answers!)

5. *(15 pts)* Compute the double integral of the function $f(x, y) = \sec^2 x$ on the region in the first quadrant with $y \leq 1$, $x \leq \pi/4$, and $y \geq \cos^2 x$.

6. *(20 pts)* Set up, but do not evaluate, a triple nested integral representing the mass of the solid tetrahedron with vertices at $(0,0,0)$, $(1,1,0)$, $(0,2,0)$, and $(0,0,4)$, where the density is given by $\delta(x,y,z) = x^2yz^3$.