EXAM 1

Math 102, 2009-2010 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ___________________________________

ID number_______________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _______________________

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________

Total Score ____________ (/100 points)

1. *(15 pts)* Find the equation of the unique plane that contains the point $(5, 5, 2)$ and is parallel to the lines parametrized by $(3 - 4t, 2t + 1, 4t)$ and $(t, 2t - 1, 3 - t)$.

2. *(15 pts)* Find the inverse of the matrix A defined by

$$A = \begin{pmatrix} 3 & -4 & 7 \\ 0 & 0 & -1 \\ -2 & 3 & 4 \end{pmatrix}$$

3. *(15 pts)* The system $A\vec{x} = \vec{b}$ relates the variables $x_1, \ldots, x_6$. The reduced row echelon form of A is the matrix below, and the variables b_1, b_2, b_3 are exogenous for this problem.

$$\text{rref}(A) = \begin{pmatrix} 1 & 2 & 0 & 6 & 4 & 0 \\ 0 & 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Which of the following sets of variables could be interpreted as endogenous in this system? (Make sure to justify each of your answers.)

(a) $\{x_1, x_3, x_4\}$

(b) $\{x_3, x_4, x_6\}$

(c) $\{x_4, x_5, x_6\}$

4. *(15 pts)* Use pivots to show that it is not possible to have a linearly independent collection of 4 vectors in $\mathbb{R}^3$. (Make sure to explain all of the steps in your reasoning.)

5. *(10 pts)* Compute the determinant of the matrix M given below.

$$M = \begin{pmatrix} 4 & 1 & -1 \\ 3 & 2 & 2 \\ 7 & 5 & 0 \end{pmatrix}$$

6. *(15 pts)* Determine if the collection $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ of vectors spans $\mathbb{R}^3$, where

$$\vec{v}_1 = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

7. *(15 pts)* Write the matrix A below as a product of elementary matrices.

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$