

EXAM 2

Math 102, 2009-2010 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name _____

ID number _____

“I have adhered to the Duke Community
Standard in completing this
examination.”

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Signature: _____

Total Score _____ (/100 points)

1. (15 pts) Find a basis for the span of the vectors below.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 2 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

2. (15 pts) The function $f : \mathbb{R}^1 \rightarrow \mathbb{R}^1$ computed by $f(x) = \sin(x)$ is not invertible. Find a way to restrict the domain and the target to form a new function $g : A \rightarrow B$ computed by $g(x) = \sin(x)$ that is invertible.

3. (15 pts) Consider the function $f(x, y, z) = xze^{x+yz+2z}$. Use the total derivative to estimate the value of $f(2.01, .02, -.99)$.

4. (20 pts) We have $z = f(x, y)$, $x = r \cos \theta$, $y = r \sin \theta$. Suppose that

$$\frac{\partial f}{\partial x}(3, 3) = 4 \quad \text{and} \quad \frac{\partial f}{\partial y}(3, 3) = 2$$

Compute $\frac{\partial z}{\partial \theta}$ at a point (r, θ) where $x = 3$ and $y = 3$.

5. (15 pts) In what direction (expressed as a unit vector) from the point $(2, 3, 1)$ is the function $f(x, y, z) = x^2y + yz^3$ increasing the fastest, and what is the directional derivative in that direction?

6. (20 pts) We consider here the surface S with equation $x^2 - y^2 = z$.

(a) Is S the graph of a function $f : \mathbb{R}^a \rightarrow \mathbb{R}^b$? If so, find a , b , and the formula for the function f .

(b) Is S a level set of a function $g : \mathbb{R}^c \rightarrow \mathbb{R}^d$? If so, find c , d , and the formula for such a function g .