EXAM 3

Math 102, 2009-2010 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

ID number____________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________

Total Score __________ (/100 points)

1. *(15 pts)* The variables w, x, y, and z are related by the equation $F(w, x, y, z) = 0$, where F is C^1. At the point (w^*, x^*, y^*, z^*) on this set, the direction of fastest increase of F is in the direction of the vector $(3, -2, 0, 4)$.

   Given this information, which of the variables can be concluded to be locally functions of the other three variables on this set? (Make sure to justify your answers!)

2. *(20 pts)* The variables p, q, r, s, and t are related by the system

$$\begin{aligned} pqr^2 - 2rs^2t &= -1 \\ p^2q^2r + 3r^2st^2 &= 4 \end{aligned}$$

Show that near the point $(p^*, q^*, r^*, s^*, t^*) = (1, 1, 1, 1, 1)$ we can view r and t as implicitly defined functions of the other variables, and compute $\frac{\partial r}{\partial p}$ at this point.

3. *(15 pts)* Let T be the solid triangle with vertices at the points $(1, 0)$, $(0, 1)$, $(0, 0)$. Find the point on T that maximizes the product of the distances from the three edges. *(Hint: Recall that the distance from a point (x, y) to the line with equation $ax + by = c$ is $d = |ax + by - c|/\sqrt{a^2 + b^2}$.)*

4. *(20 pts)* Find the point that maximizes the value of $f(x, y, z) = x + y - 3z$ subject to the constraints $z = x^2 + y^2$ and $2 \leq z \leq 8$.

5. *(15 pts)* Compute the double integral of the function $f(x, y) = xy$ over the region in the xy-plane bounded by the curves $x = y^2$ and $y = x - 2$.

6. *(15 pts)* Write a triple nested integral (but do not evaluate it!) that represents the mass of the region defined by $z \geq x^2 + y^2$ and $x^2 + y^2 + (z-1)^2 \leq 1$, where the density is given by the function $\delta(x, y, z) = e^{xyz}$.