EXAM 2

Math 102, 2010-2011 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

ID number_____

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Total Score _____ (/100 points)

1. *(pts)* In this problem we consider the functions $f, g : \mathbb{R}^2 \to \mathbb{R}^2$ defined by the formulas

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy^2 \\ y^2 - x^2 \end{pmatrix} \quad \text{and} \quad g\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy \\ e^x \end{pmatrix}$$

Suppose we wish to compose these functions as $h = g \circ f$. Use the chain rule to find the derivative matrix for this new function h.

2. *(pts)* It is known that the linear approximation to the function $P : \mathbb{R}^3 \to \mathbb{R}^2$ at the point $(3, 2, 5)$ is $L : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$L(x, y, z) = (5 - 2x + 4y + z, -3 + x - 5y + 2z)$$

Find the value $P(3, 2, 5)$, and find the derivative matrix $DP(3, 2, 5)$.

3. *(pts)* Find the equation for the tangent plane to the surface $x^2y - y^2z = xz^2 - 19$ at the point $(1, 3, 2)$.

4. *(pts)* Compute the directional derivative of the function $f(x, y) = x^2y$ at the point $(1, 2)$ in the unit vector direction $(3, 4)/5$, directly from the definition (do not use the gradient vector!).

5. *(pts)* The variables a, b, c, d, and e are related by the system

$$\begin{aligned} a^2 - (b/c) + ade^2 &= -1 \\ abc + c^2 d - e &= -2 \end{aligned}$$

Show that, near the point $(a^*, b^*, c^*, d^*, e^*) = (0, 1, 1, 0, 2)$, we can view c and d as functions a, b, and e, and compute their partial derivatives with respect to e at that point.

6. *(pts)* In this problem we consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$, evaluated by

$$f(x, y) = (e^x \cos y, e^x \sin y)$$

(a) Is this function one-to-one?

(b) Is this function onto?

(c) At what points in the domain is this function locally one-to-one?

(d) At what points in the domain is this function locally onto?