

# EXAM 3

Math 102, 2010-2011 Spring, Clark Bray.

You have 50 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING  
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

All answers must be simplified. All of the policies and guidelines  
on the class webpages are in effect on this exam.

Good luck!

Name \_\_\_\_\_

ID number \_\_\_\_\_

“I have adhered to the Duke Community  
Standard in completing this  
examination.”

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

Signature: \_\_\_\_\_

Total Score \_\_\_\_\_ (/100 points)

1. (*pts*) The expected annual profit of your business is given by the function  $\Pi(x, y, z) = xy + z^2$ , where  $x$ ,  $y$ , and  $z$  are operational parameters over which you have control, subject to the constraints that  $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ ,  $x + y + z \leq 10$ , and  $x + y = 5$ .

As CEO of this company, what choices of  $x$ ,  $y$ , and  $z$  should you choose?

*(extra space, if needed)*

2. (*pts*) Find the absolute maximizer of the function  $f(x, y, z) = 16 - x^2 - y^2 - z^2 - xy - xz - yz$ .

3. (*pts*)

- (a) Give a clear description of the domain  $D$  for which the double integral  $\iint_D e^{-x^2} dA$  is calculated by

$$\int_0^1 \int_y^1 e^{-x^2} dx dy$$

- (b) Compute the integral above by slicing in the other order.

4. (*pts*) Friendly County is in the shape of a rectangle 10 miles wide (in the east-west direction) and 5 miles tall (in the north-south direction). Locations in Friendly County are given in terms of distances  $x$  and  $y$  (measured in miles) from the eastern and southern borders (respectively).

The population density (in terms of people per square mile) of Friendly County is given by  $\delta(x, y) = 1000(x + y)$ . Every person in Friendly County says “hello” a number of times per day that is equal to the square root of the population density at his or her location.

Compute the total number of times per day that the word “hello” is uttered in Friendly County.

5. (*pts*) The probability of three random variables taking values that result in a profit for your company is given by the triple integral of the function  $e^{-x-y-z}$  over the domain defined by  $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ , and  $x + y + z = 1$ . Compute this probability.