EXAM 2

Math 103, Summer 2009 Term 1, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

Good luck!

Name ______________________________

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1. __________
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"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

Total Score __________ (/100 points)

1. *(10 pts)* Suppose that L is a linear transformation, and for certain vectors $\vec{a}$ and $\vec{b}$ we have $L(2\vec{a}+\vec{b}) = (3,2,7)$ and $L(\vec{a}+2\vec{b}) = (6,1,5)$. What is $L(\vec{a})$?

2. *(10 pts)* A bird is flying with position given by $\vec{x}(t) = (\sin t, 2t - \cos t, \sin t - \cos t)$. What is this bird's acceleration vector at the time when $t = \pi/3$?

3. *(10 pts)* For the function $f(x,y) = xy - 3y^2$ at the point $\vec{a} = (1,2)$ and with velocity $(4,5)$, compute directly from the definition the value of

$$D_{\vec{v}} f(\vec{a})$$

4. *(10 pts)* How steep is the slope of the graph of the function $f(x,y) = x^3 e^{xy} - y^3$ at the point $(3,0)$?

5. *(10 pts)* Consider the following functions.

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_2^2 \end{bmatrix} \quad \text{and} \quad g\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 + x_2 \\ x_1 x_2 \end{bmatrix}$$

Use the chain rule to write the Jacobian matrix $J_{f \circ g}$ for the composition function $f \circ g$, to be thought of as a function of the variables s and t.

6. *(10 pts)* Suppose that we have $x^2 z - xyz^2 = -2$. At the point $(x^*, y^*, z^*) = (2, 3, 1)$ on this set, what can you say about $\partial x / \partial z$?

7. *(10 pts)* Find and classify (as local max, local min, or saddle point) all of the critical points of the function $f(x, y) = x^3 - 3x - y^2$.

8. *(10 pts)* Find the point satisfying $x^2 + y^2 + z^2 \leq 1$ and $y + 2z = 1$ that maximizes the function $f(x, y, z) = x$.