EXAM 2

Math 103, 2009-2010 Summer Term 1, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
Please do not remove the staple or tear out pages.
All of the policies and guidelines on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

ID number ____________________

1. __________
2. __________
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8. __________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

Total Score __________ (/100 points)

1. *(12 pts)* The surface S is the graph of the function $f(x, y) = x^2y^2 - y^3$. The plane P has equation $12x + 5y = 65$, and the curve C is the intersection of S and P. Above the point $(5, 1)$ in the xy-plane, how steep is the curve C?

2. *(12 pts)* For the function f from the previous problem, at the point $(5, 1)$ in the xy-plane, what is the direction of fastest increase of the function?

3. *(12 pts)* Consider the following functions and variables:

$$\begin{aligned} f &: \mathbb{R}^2 \to \mathbb{R}^3, & f(p,q) &= (r,s,t) \\ g &: \mathbb{R}^3 \to \mathbb{R}^3, & g(r,s,t) &= (u,v,w) \\ h &: \mathbb{R}^3 \to \mathbb{R}^2, & h(u,v,w) &= (x,y) \end{aligned}$$

Suppose also that we know $f(\vec{a}) = \vec{b}$, $g(\vec{b}) = \vec{c}$, and

$$J_{f,\vec{a}} = \begin{pmatrix} 3 & 2 \\ 2 & 8 \\ 0 & 1 \end{pmatrix} \quad J_{g,\vec{b}} = \begin{pmatrix} 1 & -2 & 1 \\ 4 & 0 & 5 \\ -2 & 1 & 4 \end{pmatrix} \quad J_{h,\vec{c}} = \begin{pmatrix} 1 & -4 & 3 \\ -3 & 5 & 1 \end{pmatrix}$$

Compute $\dfrac{\partial y}{\partial q}(\vec{a})$

4. *(12 pts)* Consider the level set of $g(x,y,z) = xy^3 - e^y z^3 - z^2$ passing through the point $(2,0,3)$. Compute $\frac{\partial y}{\partial z}$ at this point on this surface.

5. *(13 pts)* In this problem, x and y represent miles. A field is in the shape of a triangle with vertices at $(0,0)$, $(3,3)$, and $(0,6)$. The population density of mice in the field (in units of thousands of mice per square mile) is $p(x,y) = 3x + 2y$. Compute the total population of mice in the field.

6. *(13 pts)* The region R in xyz-space is bounded by the surfaces $x = 0$, $x = y$, $x + y = 2$, $(x+1)(y+1)(z+1) = 3$, and $x + y + z = 0$. The mass density in that region is given by $\delta(x,y,z) = e^{xyz}$. Write down, but do not evaluate, a triple nested integral that represents the mass inside R.

7. *(13 pts)* The rectangle R has vertices at $(-1,1)$, $(1,1)$, $(-1,4)$, $(1,4)$ in the uv-plane. Its image by the function $f(u,v) = (3u+6v, 2v) = (x,y)$ is a parallelogram P in the xy-plane. Compute $\iint_P (x-3y)^7 y^4 \, dx \, dy$.

8. *(13 pts)* Set up, but do not evaluate, a triple nested integral in spherical coordinates that represents $\iiint_R (x^2+y^2+z^2) \, dV$, where R is the ball of radius $\sqrt{2}$ centered at $(1,1,0)$.