EXAM 2

Math 103, 2010 Summer Term 2, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________________

ID number_________________________

1. _____________
2. _____________
3. _____________
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5. _____________
6. _____________
7. _____________
8. _____________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _________________________

Total Score _____________ (/100 points)

1. *(12 pts)* We consider here $f : \mathbb{R}^2 \to \mathbb{R}^1$, which is differentiable at the point $\vec{a}$. Suppose that the unit directional derivative of f at $\vec{a}$ is maximized in the direction indicated by the vector $(1, 2, 4)$, and that the directional derivative of f at $\vec{a}$ with velocity $(3, 5, 1)$ is 12. Compute the directional derivative of f at $\vec{a}$ with velocity $(2, -2, 1)$.

2. *(12 pts)* We consider here differentiable functions $g : \mathbb{R}^3 \to \mathbb{R}^3$, $f : \mathbb{R}^3 \to \mathbb{R}^3$, and $h : \mathbb{R}^3 \to \mathbb{R}^3$, with h defined by $h(u, v, w) = f(g(u, v, w))$. Suppose we know that $g(\vec{a}) = \vec{b}$, and that

$$J_{g,\vec{a}} = \begin{pmatrix} 7 & 2 & 1 \\ -2 & 4 & -1 \\ 0 & 3 & -3 \end{pmatrix} \quad \text{and} \quad J_{f,\vec{b}} = \begin{pmatrix} 5 & -2 & 6 \\ 1 & 2 & 1 \\ -4 & 4 & 1 \end{pmatrix}$$

Compute $\frac{\partial h_3}{\partial u}$.

3. *(12 pts)* Suppose we know that $x^3e^{yz} - xze^{xy} + z^3 = 1$. Near the point $(1, 1, 1)$, what is $\frac{\partial x}{\partial z}$?

4. *(12 pts)* Compute the value of the integral $\iiint_R z\, dV$, where R is the region in $\mathbb{R}^3$ above the xy-plane and inside the unit sphere.

5. *(12 pts)* Compute the value of the integral $\iint_D x^5\,dx\,dy$, where D is the region in the xy-plane bounded by the curves $x+y-x^3=1$, $x+y-x^3=-1$, $x+y=2$, and $x+y=-3$.

6. (a) *(8 pts)* Write down, but do not evaluate, a nested integral that represents the value of the triple integral $\iiint_R x^3yze^{(x^2y)}\,dV$, where R is the region in $\mathbb{R}^3$ above the unit disk in the xy-plane and below the surface $z = x^2 + y^2$.

   (b) *(8 pts)* Evaluate this triple integral, using any means from this course.

7. *(12 pts)* A certain very weak and heavy spring, compressed under its own weight, follows the path parametrized by $\vec{x}(t) = (\cos t, \sin t, e^t)$, for $t \in [0, 4\pi]$. The amount of paint on the side of the spring (mass of paint per unit length on the spring) is equal to $z^2/100$. Compute the total mass of the paint on the spring.

8. *(12 pts)* A metal sheet is on the surface S, which is the part of the plane $2x + y + z = 10$ above the rectangle $[0, 1] \times [1, 3]$ in the xy-plane. The mass per unit area on this sheet is $\delta(x, y, z) = z$. Compute the total mass of this sheet.