EXAM 1

Math 103, 2012 Summer Term 1, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name _____

1. _____

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"I have adhered to the Duke Community
Standard in completing this
examination."

Signature: _____

Total Score _____ (/100 points)

1. *(10 pts)* Bob is walking along a sidewalk on a windy day. Due to the wind, he has to exert a force given by the vector $\vec{F} = (3,6)$. If his total displacement while walking on the sidewalk is given by the vector $\vec{d} = (-12, 7)$, what is the total amount of work that he performs in this process?

2. *(12 pts)* The list $\vec{e}_1, \vec{v}, \vec{w}$ is in right-hand order, and these three vectors define a parallelepiped whose volume is equal to 12. The list $\vec{e}_2, \vec{v}, \vec{w}$ is in left-hand order, and these three vectors define a parallelepiped whose volume is equal to 7. The list $\vec{e}_3, \vec{v}, \vec{w}$ is in left-hand order, and these three vectors define a parallelepiped whose volume is equal to 4.

   Compute the volume of the parallelepiped defined by the vectors $\vec{a}, \vec{v}, \vec{w}$ (where $\vec{a} = (2, 1, 3)$), and decide if that list is in left-hand order or right-hand order.

3. *(12 pts)* Find the spherical equation for the sphere of radius three centered at the point $(0, -3, 0)$.

For problems 4-7, we consider the surface S in $\mathbb{R}^3$ with equation $z(x^4 + 2x^2y^2 + y^4) = 1$.

4. *(7 pts)* Is this surface S the graph of a function f? If so find a formula for such a function, and its domain and target.

5. *(7 pts)* Is this surface S a level set of a function g? If so, find a formula for such a function, and its domain and target.

6. *(8 pts)* Is this surface S parameterized by some function h? If so, find a formula for such a function, and its domain and target.

7. *(8 pts)* Is this surface S a rotation of some curve in one of the coordinate half planes? If so, indicate which coordinate half plane, which axis it is rotated around, and the equation for that curve within that half plane.

8. *(12 pts)* Compute this limit, if it exists, or explain how you know it does not exist.

$$\lim_{\vec{x} \to \vec{0}} \frac{x^3 y - xy^2}{x^3 - y^3}$$

9. *(12 pts)* The linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^3$ has $L(\vec{e}_1) = (2, 3, 1)$ and $L(\vec{e}_2) = (8, 2, 4)$. Find an explicit formula for $L(x, y)$, and also find the matrix M representing L.

10. *(12 pts)*

   (a) Find an expression $\vec{x}(t)$ for the parametric line that passes through the point $\vec{a} = (3, 2)$ with velocity equal to $\vec{w} = (1, -3)$.

   (b) Find an expression for the parametric curve $\vec{c}(t)$ which is the image of that parametric line (from part (a)) by the function f defined by $f(x, y) = (x^2 y, 5xy)$.

   (c) Find an expression for the velocity $\vec{v}(t)$ of this parametric curve (from part (b)).

   (d) What is the relationship between the velocity $\vec{v}(t)$ and the directional derivative $D_{\vec{w}} f(\vec{a})$? Use this relationship to compute the directional derivative.

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