EXAM 2

Math 212, 2016 Summer Term 2, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ____________________

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________

Total Score __________ (/100 points)

1. *(15 pts)* Bob is in a field at the location $(2, 1)$, and he knows that the density of flowers (in blooms per square meter) near where he is in the field is given by $\delta(x, y) = xy + x^2$.

   (a) If he were to walk in the direction of the vector $(3, 4)$, what would be the rate of change per unit distance travelled of the density function δ?

   (b) Which direction should he walk if he wants δ to increase as quickly as possible?

   (c) In terms of the rate of change of flower density, by what proportion is the direction from part (b) better than the direction from part (a)?

2. *(15 pts)* The function f is defined by

$$f(x,y) = \begin{cases} xe^{(y^2/x^2)} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) Show that f is directionally differentiable at $(0,0)$ by finding a general expression for $D_{\vec{v}}f(0,0)$ in terms of $\vec{v}$.

(b) Is f differentiable at $(0,0)$? Explain your reasoning.

3. *(15 pts)* The following is given about various functions and variables:

$$f(r,s,t) = (x,y,z) \qquad g(x,y,z) = (a,b,c) \qquad f(1,5,3) = (2,4,6)$$

$$J_{f,(1,5,3)} = \begin{pmatrix} 1 & 5 & -1 \\ -1 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix} \qquad J_{g,(2,4,6)} = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 6 & -1 \\ 4 & -2 & 2 \end{pmatrix}$$

Suppose that the variable c is set constant. Does this allow for interpreting t as a function of r and s near the point $(r,s,t) = (1,5,3)$?

4. *(15 pts)* The region R in the xy-plane is bounded by the x-axis and the curves $y = x^3$ and $x + y = 2$. Compute the integral over R of the function $f(x,y) = x + y$.

5. *(15 pts)* The domain D in $\mathbb{R}^3$ is described by $x^2 + y^2 + z^2 \leq 4$, $z \leq 1 - x^2$, and $z \geq 0$. Set up, but do not evaluate, a triple nested integral in rectangular coordinates that represents the y-coordinate of the centroid of D. (You may leave the volume of this domain D as V.)

6. *(10 pts)* Your friend Bob says that the value of the integral from question 5. (above) is zero, by symmetry. Is he right or wrong? Explain fully.

7. *(15 pts)* The sphere S has radius 1 and center at $(0, 0, 1)$; the cone C has equation $z = \sqrt{x^2 + y^2}$. The domain T in $\mathbb{R}^3$ is the region inside of S and above C. Set up, but do not evaluate, a triple nested integral in spherical coordinates representing the mass in T given that the density in that region is given by $\delta = \delta(x, y, z)$.