EXAM 1

Math 212, 2017 Summer Term 2, Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.
All answers must be simplified. All of the policies and guidelines
on the class webpages are in effect on this exam.

Good luck!

Name ______________________________

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ______________________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

Total Score __________ (/100 points)

1. *(20 pts)* The pilot of an airplane directly controls the "air velocity" ($\vec{v}_a$) of the plane (the velocity of the plane relative to the air it is moving in), but gets to the desired destination by understanding the "ground velocity" ($\vec{v}_g$) of the plane (the velocity of the plane relative to the ground). Their relationship involves the "wind velocity" ($\vec{v}_w$).

   (a) What is the algebraic relationship between $\vec{v}_a$, $\vec{v}_g$, and $\vec{v}_w$?

   (b) The pilot needs a ground velocity of 200 miles per hour due east to get to the destination on time, and the wind is blowing a constant $20\sqrt{2}$ miles per hour directly northeast. Compute the air velocity vector that would be necessary to get the plane to the destination on time.

   (c) Is the "air speed" (the magnitude of the air velocity) from part (b) greater or less than it would have to be if there were no wind at all?

   (d) How fast would the wind have to blow (still directly northeast) in order to allow the plane to achieve the same ground velocity with as little air speed as possible?

2. *(18 pts)*

(a) A plane P contains the points $\vec{p}$ and $\vec{q}$, and is parallel to the vector $\vec{v}$. Find an equation for P in terms of these givens.

(b) The above equation can be written as

$$\det \begin{pmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{pmatrix} = 0$$

Find vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ that work for this purpose.

3. *(20 pts)* Your friend Bob is trying to understand a certain surface S in xyz-space. He knows only that the cross section of this surface in any plane P parallel to the xz-plane has equation

$$z^2 - x = -4k^2$$

where k is the distance from P to the xz-plane.

(a) Find an equation for S.

(b) Find an equation for the surface T obtained by stretching S by a factor of 2 in the y-direction.

(c) Give clear geometric descriptions for T and S.

4. *(20 pts)* S is the graph of the function $f : \mathbb{R}^2 \to \mathbb{R}^1$ given by $f(x, y) = (\sin x)e^{-(x^2+y^2)}$.

(a) Find a function h for which one of the levels sets is S, and identify the domain and the target of h.

(b) Find a function $\vec{x}$ that parametrizes S, and identify the domain and the target of $\vec{x}$.

5. *(8 pts)* Compute the following limit or show that it does not exist.

$$\lim_{\vec{x}\to\vec{0}} \frac{x}{x^2+y^2}$$

6. *(14 pts)* $L : \mathbb{R}^3 \to \mathbb{R}^3$ is a rotation of $\mathbb{R}^3$ around the origin, with $L(1,0,0) = (\frac{6}{7}, \frac{2}{7}, \frac{-3}{7})$ and $L(1,1,0) = (\frac{8}{7}, \frac{5}{7}, \frac{3}{7})$. Find the matrix A representing L. *(Hints: (1) L is a linear transformation; (2) Every rotation R preserves cross products: $\vec{u} = \vec{v} \times \vec{w} \Longrightarrow R(\vec{u}) = R(\vec{v}) \times R(\vec{w})$.)*