

# EXAM 1

Math 212, 2018 Summer Term 2, Clark Bray.

Name: \_\_\_\_\_ Section: \_\_\_\_\_ Student ID: \_\_\_\_\_

## GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT.  
CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators. Scratch paper is allowed, but (1) it must be from the instructor, (2) it must be returned with the exam, and (3) it will NOT be graded.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

## WRITING RULES

Do not write anything on the QR codes or near the staple.

Use black pen only – no pencils.

Work for a given question can be done ONLY on the front or back of the page the question is written on.

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## DUKE COMMUNITY STANDARD STATEMENT

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: \_\_\_\_\_

(Nothing on this page will be graded!)

1. (20 pts) The line  $L \in \mathbb{R}^3$  is parametrized by  $\vec{x}(t) = (3 - t, 2t, t + 1)$ , and the line  $C$  has symmetric equations

$$\frac{x - 1}{2} = \frac{y + 1}{6} = \frac{z + 3}{10}$$

- (a) Do  $L$  and  $C$  intersect? If so, find the point of intersection; if not, explain how you know.

- (b) Find the equation of the plane that contains  $L$  and is parallel to  $C$ .

*(extra space for question from other side)*

2. (20 pts)

(a) The point  $(x, y, z) = (0, 1, 1)$  can be described in spherical coordinates in exactly one way with  $\pi \leq \theta \leq 2\pi$  and  $-\pi \leq \phi \leq 0$ . Find these values of  $\rho$ ,  $\phi$ , and  $\theta$ .

(b) Find the spherical coordinates equation for the sphere of radius 1 centered at  $(0, -1, 0)$ .

*(extra space for question from other side)*

3. (20 pts)

(a) The surface  $S$  has equation  $\sqrt{x^2 + z^2} = 1 - |y|$ . Give a clear description of simple geometric processes that would generate  $S$ .

(b) Give an explicit, ordered series of geometric transformations that can be applied to the unit sphere to result in the surface with equation  $4x^2 + (4y + 8)^2 + z^2 = 1$ .

*(extra space for question from other side)*



4. (20 pts) The surface  $S$  has equation  $xe^y - ze^{xy} = ye^x$ .

(a) Find a function  $p : \mathbb{R}^a \rightarrow \mathbb{R}^b$  whose graph is  $S$ , and identify the values of  $a$  and  $b$ .

(b) Find a function  $q : \mathbb{R}^c \rightarrow \mathbb{R}^d$  for which a level set is  $S$ , and identify the values of  $c$  and  $d$ .

(c) Find a function  $r : \mathbb{R}^f \rightarrow \mathbb{R}^g$  that parametrizes  $S$ , and identify the values of  $f$  and  $g$ .

*(extra space for question from other side)*

5. (20 pts) The function  $L$  is a linear transformation, and we are given

$$L\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad \text{and} \quad L\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$$

Find the matrix  $A$  that represents  $L$ . (*Hint: Start by writing  $(0, 1)$  as a linear combination of  $(1, 0)$  and  $(1, 1)$ .*)

*(extra space for question from other side)*