

# EXAM 2

Math 212, 2018-2019 Summer Term 1 (Marine Lab), Clark Bray.

You have 75 minutes.

No notes, no books, no calculators.

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING  
TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

All answers must be simplified. All of the policies and guidelines  
on the class webpages are in effect on this exam.

Good luck!

Name \_\_\_\_\_

“I have adhered to the Duke Community  
Standard in completing this  
examination.”

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

Signature: \_\_\_\_\_

Total Score \_\_\_\_\_ (/100 points)

1. (20 pts) The function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$  is given by  $f(x, y) = xy - 3x$ .

(a) Compute  $\left. \frac{d}{dt} \right|_{t=0} f(1 - 2t, t + 3)$  *without* using the differentiability of  $f$ .

(b) Use the result above to compute the slope of the graph of  $f$  at  $(1, 3)$  in the direction indicated by  $(-2, 1)$ .

(c) Use the result of either of the above to compute the rate of change of  $f$  experienced by a particle that is at the point  $(1, 3)$  and moving in the direction indicated by  $(-2, 1)$  with velocity equal to 5.

2. (20 pts) The function  $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by

$$g \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 - 5y + y^2 \\ e^{xy} + \sin(x) \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$$

(a) Compute the directional derivative of  $g$  at the point  $(0, 1)$  with velocity  $(2, 3)$ . Be sure to identify and justify any important properties of  $g$  that you might use in your calculation.

(b) Find the equation of the tangent plane to the graph of the coordinate function  $g_2$  above the point  $(0, 1)$ .

3. (20 pts)

(a) Set up but do not evaluate a triple iterated integral representing  $\iiint_R f \, dV$ , where  $f = x^3y^4$  and  $R$  is the solid bounded by the surfaces  $z = e^{8-x^2-y^2}$  and  $z = e^{x^2+y^2}$ .

(b) Compute the above integral using any method from this course.

4. (20 pts) The ball  $B$  is centered at  $(-1, 0, 0)$  with radius 1, and the domain  $D$  is the part of  $B$  with  $z \leq 0$ . Set up but do not evaluate a triple iterated integral in spherical coordinates representing the mass in  $D$  where density is given by  $\delta(x, y, z) = z^2$ .

5. (20 pts) The potential energy of a mass  $m$  at height  $h$  above the  $xy$ -plane is given by  $E = mgh$ , where  $g$  is a known constant.

A strip of heavy rubber is hanging stretched around a cylinder, following the curve  $C$  parametrized by  $\vec{x}(t) = (\cos 6t, \sin 6t, e^t)$ ,  $t \in [0, 2]$ . The mass of rubber per unit length on the curve is  $\delta = k/\sqrt{z}$  (where  $k$  is a constant). Write down but do not evaluate a single variable integral in terms of  $t$  that represents the total potential energy of the rubber.