

# EXAM 1

Math 212, 2020 Summer Term 1, Clark Bray.

Name: \_\_\_\_\_ Section: \_\_\_\_\_ Student ID: \_\_\_\_\_

## GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT.  
CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

## WRITING RULES

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

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## DUKE COMMUNITY STANDARD STATEMENT

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: \_\_\_\_\_

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1. (20 pts) A solar cell is in the shape of a rectangle with edge vectors  $(2, 2, 0)$  and  $(1, -1, 1)$ , and one corner at the origin. (Of course it is facing toward the sky, not toward the ground.)

(a) Find the equation of the plane containing the solar cell.

(b) The “efficiency of the orientation” is the cosine of the angle between the “orientation vector” of the cell (unit vector perpendicular to the cell in the direction it is facing) and the direction of the sun. Compute the efficiency of this cell’s orientation at the moment when the sun is in the direction of  $\theta = \pi/2$  and  $\phi = \pi/4$ .

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2. (15 pts) Find the spherical equation of the sphere of radius  $\sqrt{2}$  centered at  $(1, 1, 0)$ .

3. (5 pts) Find the rectangular equation of the surface with spherical equation  $\rho = \csc \phi \sec \theta$ .

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4. (5 pts) The function  $m : \mathbb{R}^3 \rightarrow \mathbb{R}^1$  is given by  $m(x, y, z) = 2x + 3y - z$ , and the function  $p : \mathbb{R}^2 \rightarrow \mathbb{R}^1$  is given by  $p(x, y) = 2x + 3y$ . There is a relationship between these functions involving their graphs and/or level sets; express that relationship in a single brief sentence. (Be especially careful with your words – this question is about the proper use of these terms!)
5. (15 pts) A child standing at  $(0, 4)$  in the  $xy$ -plane has a ball attached to a string, and she is swinging the ball over her head counterclockwise in a horizontal circle of radius 3 feet, making one complete revolution per second. At the moment ( $t = 0$ ) when the ball is at  $(3, 4)$ , she starts running in the  $x$ -direction with speed 5 feet per second, while continuing to swing the ball. Find the position and the velocity of the ball as functions of time.

*(extra space for questions from other side)*



6. (5 pts) Bob is told that  $T$  is a linear transformation, and that  $T(\vec{e}_1) = (1, 2, 3)$ ,  $T(\vec{e}_2) = (4, 5, 6)$ , and  $T(\vec{e}_3) = (7, 8, 9)$ . He then computes  $T(\vec{e}_1 + \vec{e}_3)$  as

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 10 \\ 16 \end{pmatrix} \quad \text{and also as} \quad T(\vec{e}_1) + T(\vec{e}_3) = \begin{pmatrix} 8 \\ 10 \\ 12 \end{pmatrix}$$

Can you help Bob resolve this seeming contradiction?

7. (15 pts) Reflection over the line  $L$  in the  $xy$ -plane defined by the polar angle  $\theta$  can be achieved by first rotating clockwise by the angle  $\theta$ , then reflecting over the  $x$ -axis, and then rotating counterclockwise by the angle  $\theta$ . Use this information to find the matrix representing reflection over the line  $L$ .

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8. (20 pts) We have  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$ , and  $D_{\vec{v}}f(\vec{a}) = 3$ , where  $\vec{v} = (2, 3)$ . We consider here a particle at  $\vec{a}$  moving with velocity  $\vec{v}$ .

(a) Compute  $df/dt$ , the rate of change of  $f$  with respect to time  $t$ .

(b) Compute  $df/ds$ , the rate of change of  $f$  with respect to distance traveled  $s$ .

(c) Suppose the particle were moving the same direction, but with speed equal to 5; what would be the rate of change of  $f$  with respect to time  $t$ ?

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