

EXAM 2

Math 212, 2020 Summer Term 1, Clark Bray.

Name: _____ Section: _____ Student ID: _____

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT.
CLARITY WILL BE CONSIDERED IN GRADING.

No calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

DUKE COMMUNITY STANDARD STATEMENT

“I have adhered to the Duke Community Standard in completing this examination.”

Signature: _____

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1. (10 pts) Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $f(x, y) = (x + |y|, y)$. Is f directional linear at the origin? Prove or disprove.

2. (10 pts) The function $g : \mathbb{R}^2 \rightarrow \mathbb{R}^1$ is differentiable, and for the vectors $\vec{v} = (1, 2)$ and $\vec{w} = (0, 2)$ we know the directional derivatives $D_{\vec{v}}g(\vec{a}) = 5$ and $D_{\vec{w}}g(\vec{a}) = 3$. Find the Jacobian matrix $J_{g, \vec{a}}$.

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3. (10 pts) Suppose that x and y are the usual functions of the polar coordinates r and θ , and w is continuously twice differentiable function of x and y . Find a fully simplified expression for

$$\frac{\partial}{\partial \theta}(w_x w)$$

4. (10 pts) The function $f : \mathbb{R}^3 \rightarrow \mathbb{R}^1$ is continuously differentiable, and we know that at the point \vec{a} we have $\nabla f(\vec{a}) = (2, 1, 0)$.

On the level set passing through \vec{a} , decide which of $\frac{\partial x}{\partial y}$ and $\frac{\partial z}{\partial x}$ must exist and compute if possible.

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5. (15 pts) The domain D is bounded by the surfaces $x - 2y^2 - z^2 = 0$ and $x + y^2 + 2z^2 = 12$, and mass is distributed through D with density $\delta = e^x$. Set up, but do not evaluate, a triple nested integral representing $\iiint_D e^x dV$.

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6. (25 pts) The ellipsoids in xyz -space have equations $x^2 + y^2 + 4z^2 = 4$ and $x^2 + y^2 + 4z^2 = 9$. The solid region between these two ellipsoids is R . Compute the integral over R of the function $g(x, y, z) = x^2 + y^2 + 4z^2 - x - 1$. (Hint: Use a change of variables to turn the domain into a more convenient shape.)

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7. (20 pts) The plane P has equation $3x - 2y + 5z = 1$, and the surface S is the part of P with $0 \leq y \leq 1$ and $0 \leq z \leq 2$. Compute the integral over S of the function $f(x, y, z) = y$.

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