

- (20 pts) We consider here the points $\vec{a} = (1, 0, 0)$, $\vec{b} = (0, 1, 0)$, $\vec{c} = (0, 0, 1)$, $\vec{k} = (3, 3, 6)$. (A) Find the magnitude of the vector \vec{d} represented by the arrow with tail at \vec{a} and head at \vec{k} . (B) Find a vector \vec{p} perpendicular to the plane P containing \vec{a} , \vec{b} , \vec{c} . (C) Find the equation of the plane Q that is parallel to P and contains \vec{k} . (D) Find $\vec{d} \cdot \vec{p}$ as part of a computation of the distance from \vec{k} to P .
- (20 pts) (A) Identify a standard curve of high school algebra and an explicit, ordered sequence of geometric transformations on it by which you can produce the surface S with equation $4x^2 + z^2 = e^{2y-3}$. (B) Cross sections of S parallel to one of the coordinate planes give ellipses; identify which coordinate plane this is.
- (20 pts) (A) The surface S has equation $xe^y + y^2z = z^3$. Find functions f , g , and h , along with their domains and targets, for which S is the graph, a level set, and an image (respectively). (B) Parametrize the curve that is the intersection of S with the cylinder $y^2 + z^2 = 9$.
- (20 pts) T is a linear transformation from \mathbb{R}^3 to \mathbb{R}^3 , and we know $T(1, 4, 2) = (3, 2, 5)$, $T(0, 1, 2) = (1, 0, 1)$, $T(0, 0, 1) = (1, 1, 1)$. (A) Find the matrix representing T . (B) Find the matrix representing $T \circ S$, where S reflects vectors through the plane $x = y$.
- (20 pts) Compute *directly from the definition* the directional derivative $D_{\vec{v}}g(\vec{a})$, where $\vec{a} = \vec{0}$, $\vec{v} = (2, 3)$, and $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by $g(x, y) = (xe^y, y \sin x)$.