1. *(20 pts)* We consider here the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = \frac{xy^2 - x^3}{x^2+1}$. (A) Show that f is continuously differentiable. (B) What is the most immediate argument to show that f is also differentiable? (C) Use the most efficient argument to compute the unit directional derivative of f at the point $(1,2)$ in the direction from there toward the point $(5,5)$. (D) In what direction from $(1,2)$ is f increasing the most quickly?

2. *(20 pts)* Suppose f and g are differentiable, with $f(x,y) = (u,v)$ and $g(u,v) = (s,t)$, and $f(3,2) = (5,3)$, and $J_{f,(3,2)} = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}$ and $J_{g,(5,3)} = \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}$. (A) Compute $\frac{\partial s}{\partial y}$. (B) Bob says he would like to hold s constant and compute $\frac{dy}{dx}$; give an appropriate argument to help Bob as best you can.

3. *(15 pts)* The solid domain D is defined by $x + y + z \leq 3$, $x \geq 0$, $y \geq 0$, $y + z \leq 2$, $y \leq z$. Write $\iiint_D e^x \, dV$ as a triple nested integral (but do not evaluate!).

4. *(25 pts)* The ball B is centered at the origin with radius 3. The solid domain D is the part of B above both $y = z$ and $y = -z$. We are interested in computing the integral $I = \iiint_D z \, dV$. (A) Use a rotation as a change of variables to relate I to an integral over a new domain Q that would be more convenient for use with spherical coordinates. (B) Write the integral over Q from (A) in spherical coordinates, and compute I.

5. *(20 pts)* The curve C is the first quadrant portion of the circle centered at $\vec{0}$ of radius 3. (A) Write a single variable integral (but do not compute!) representing $\int_C x^2 \, ds$. (B) Use symmetry to show that $\int_C x^2 - y^2 \, ds = 0$ and thus $\int_C x^2 \, ds = \int_C y^2 \, ds$. (C) Use the result of (B) and the fact that $x^2 + y^2$ is constant on C to compute $\int_C x^2 \, ds$ <u>without</u> directly computing the single variable integral from (A).