

- (24 pts) The function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is defined by $f(x, y) = (xy^3, ye^x, x + y) = (f_1, f_2, f_3)$. (A) Show that f is differentiable. (B) Compute $D_{\vec{v}}f(\vec{a})$ with $\vec{a} = (0, 2)$ and $\vec{v} = (1, 3)$. (C) Considering the surface with equation $z = f_2(x, y)$ at the point where $x = 0$ and $y = 2$, in what direction is z increasing the fastest, and how steep is the surface in that direction?
- (20 pts) The functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $g : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ are defined by $f(x, y) = (xy, x + y)$ and $g(x, y) = (2xy, x^2 - xy, x^3)$. (A) Without composing these functions directly, find the Jacobian matrix for the composition $h(x, y) = (g \circ f)(x, y) = (h_1, h_2, h_3)$ at the point $(1, 2)$. (B) Does the curve with equation $h_2(x, y) = h_2(1, 2)$ allow for viewing y as a function of x near $(1, 2)$?
- (16 pts) The region R is the tetrahedron with vertices at $(0, 0, 1)$, $(0, 0, 2)$, $(1, 1, 0)$, and $(-1, 1, 0)$. Mass is distributed through R with density $\delta(x, y, z) = 1 + \sin(xyz)$. Write (but do not evaluate) a triple nested (iterated) integral representing the mass in the region R .
- (24 pts) In uvw -space, the line segment S goes from $(0, 0, 0)$ to $(1, 2, 3)$. The solid M has horizontal ($w = c$) cross sections that are disks of radius 2 with their centers all on S . The image of M by the linear transformation $L(u, v, w) = (u - \frac{1}{3}w, v - \frac{2}{3}w, w) = (x, y, z)$ is a right cylinder R in xyz -space whose axis is the z -axis. (A) Compute the Jacobian determinant of L . (B) Compute $\iiint_M u \, du \, dv \, dw$.
- (16 pts) The hyperbolic paraboloid H has equation $z = 6 + 2y^2 - x^2$. The solid ellipse E in the xy -plane is described by $x^2 + 4y^2 \leq 4$. A potato chip is in the shape P that is the part of H sitting above E . (A) Write (but do not yet evaluate) a double nested (iterated) integral representing the area of P . (Bonus (10 pts):) Compute the area of P .