Math 221 Homework for Section 4.3: additional problems on change of basis

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Date: due Tuesday 9 November 2021

1. Let V be an inner product space of dimension 7 with a subspace W of dimension 2. Fix linearly independent $\mathbf{w}_1, \mathbf{w}_2 \in W$ and linearly independent $\mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6, \mathbf{v}_7 \in V$ all orthogonal to W. Then $\mathcal{B} = \mathbf{w}_1, \mathbf{w}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6, \mathbf{v}_7$ is a basis for V (why?). Use the general formula
$$T\begin{bmatrix} \mathbf{x}_1 & \cdots & \mathbf{x}_n \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 & \cdots & \mathbf{x}_n \end{bmatrix} \begin{bmatrix} T \end{bmatrix}_{\mathcal{B}}$$
with $T = \mathrm{proj}_W$ to find the 7×7 matrix $\begin{bmatrix} \mathrm{proj}_W \end{bmatrix}_{\mathcal{B}}$ for the orthogonal projection of V onto W with respect to $\mathcal{B}$. Is anything simpler if $\mathbf{w}_1$ and $\mathbf{w}_2$ are orthogonal or orthonormal? Hint: which of these basis vectors does proj_W fix? What happens to the others?

2. In Question 1, suppose $V = \mathbb{R}^7$ and
$$\mathbf{w}_1 = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 0 \\ 0 \\ 3 \\ -1 \end{bmatrix} \text{ and } \mathbf{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$
Let $A = \begin{bmatrix} \mathrm{proj}_W \end{bmatrix}_{\mathcal{E}}$ be the matrix of the projection onto W with respect to the standard basis. Express A as a product of matrices and their inverses; do not attempt to invert or multiply the matrices.

3. The polynomial $f(t) = 6 - 2t^2 + t^3 - \pi t^4$ can be expressed as a matrix product
$$f(t) = \begin{bmatrix} 1 & t & t^2 & t^3 & t^4 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ -2 \\ 1 \\ \pi \end{bmatrix}.$$
This polynomial $f(t)$ is a linear combination of the polynomials in the row vector $\begin{bmatrix} 1{+}t{+}t^2 & t{+}t^2{+}t^3 & t^2{+}t^3{+}t^4 & t^3{+}t^4 & t^4 \end{bmatrix}$. The coefficients in this linear combination are the entries of a column vector of size 5. Express that column as the product of a matrix and the column vector displayed above; do not attempt to invert or multiply any matrices (unless you'd like to check your answers!). Hint: additional problem 3 from Section 4.4.