SUBWORD COMPLEXES IN COXETER GROUPS

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Abstract. Let (Π, Σ) be a Coxeter system. An ordered list of elements in Σ and an element in Π determine a *subword complex*, as introduced in [KM03]. Subword complexes are demonstrated here to be homeomorphic to balls or spheres, and their Hilbert series are shown to reflect combinatorial properties of reduced expressions in Coxeter groups. Two formulae for double Grothendieck polynomials, one of which appeared in [FK94], are recovered in the context of simplicial topology for subword complexes. Some open questions related to subword complexes are presented.

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