Introduction to Partial Differential Equations
Math 232, Spring 2012

Instructor: Jim Nolen
Email: nolen@math.duke.edu
Office: Physics 243
Office Hours:

Course Description: This is an introductory, graduate-level course on Partial Differential Equations (PDEs). Many of the fundamental laws of nature are formulated as partial differential equations. Their qualitative behavior is as varied as the world around us. This course introduces some of the main types of PDEs (primarily first order and second order PDEs) and their mathematical treatment. A tentative schedule of specific topics covered is given below.

Prerequisites: A solid foundation in multivariable calculus and ordinary differential equations. A course in real analysis is strongly recommended.

Text: I will post some detailed lecture notes on the class website in Sakai. The following texts will be on reserve in the library:

1. L.C. Evans, *Partial Differential equations*, American Mathematical Society, second edition, 2010.
2. Y. Pinchover and J. Rubinstein, *An Introduction to Partial Differential Equations*, Cambridge University Press, 2005.
3. W.A. Strauss, *Partial Differential Equations: An Introduction*, Wiley, 1992.

Evaluation: Your course grade will be determined by:

- **Regular problem sets.** (70%) Your solutions will be written and turned in. For some problems, you will be asked to present solutions orally.
- **Final Exam.** (30%) The exam will be comprehensive.

Miscellaneous:

- You are encouraged to discuss the homework problems and their solution with your classmates. However, the work that you turn in must represent your own effort.
- Do as many exercises as you can, not just those that are assigned.
- Attend the Applied Math/Analysis Seminar (usually on Mondays at 4:30). The speakers often will talk about PDEs that we will study (or closely-related equations).
- Notes, homework, and other materials will be posted on the course website in Sakai:
https://sakai.duke.edu

Tentative Schedule of topics:

1. First order equations
   - Method of characteristics for linear and quasilinear equations
   - Transport equation
   - Burgers' equation and shocks
   - Conservation laws and weak solutions
   - Rankine-Hugoniot condition, entropy solutions
2. Intro second order linear PDE
   - Derivation of Laplace, Poisson, wave and heat equations.
3. The wave equation for dimension $d = 1, 2, 3$
   - D'Alembert's formula for $d = 1$
   - Kirchoff's formula via spherical means for $d = 3$
   - Method of descent for $d = 2$
   - Domain of dependence, finite speed of propagation, Huygen's principle
   - Energy conservation, causality
   - Boundary value problems on the half line
   - Sources, Duhamel's principle
4. Boundary value problems, separation of variables, and Fourier Series
   - Boundary conditions on bounded domains
   - Separation of variables
   - Fourier Series.
   - Inhomogeneous terms, inhomogeneous boundary conditions
   - General eigenfunction expansion.
5. Laplace and Poisson equations
   - Harmonic functions: maximum principle, mean value property, regularity
   - Fundamental solutions, Green's functions
   - Poisson formula for a ball in $\mathbb{R}^d$
6. Heat equation
   - Fundamental solution and convolution formula
   - Properties of solutions: maximum principle, regularity
   - Sources and Duhamel's principle
   - Boundary value problem on the half-line
   - Probabilistic interpretation

7. The Fourier Transform
   - Definition and properties of the Fourier transform.
   - Applications of the Fourier transform to PDEs
8. Weak Derivatives and Sobolev Spaces
   - Distributions and weak derivatives.
   - Sobolev Spaces and their properties.
   - Variational (weak) form of elliptic boundary value problems.
9. Eigenvalues of elliptic operators, and variational methods
   - Dirichlet's principle
   - Variational characterization of eigenvalues, eigenfunctions
10. Introduction to numerical methods for PDEs.