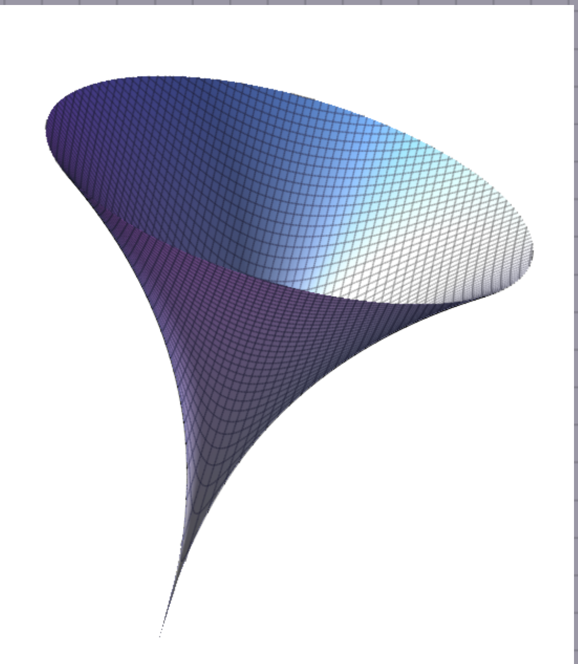
Look at an ice cream cone or a horn (like the one pictured); if you slice it horizontally, you'll get a circle. As you slice closer to the tip, the circles get smaller and smaller. The difference between the cone and the horn is that in the horn the circles get smaller faster. You can make other shapes by using cross-sections other than circles: you could use spheres or tori, or higher dimensional things. For these shapes, the tip is a singularity, but it is a special kind of singularity that we can say interesting things about. It turns out that if we plot the solutions to systems of algebraic equations like $z^3 = x^2 + y^2$, the singularities you get can often be thought about this way. We can use tools like analysis to answer questions about how the topology of these cross-sections morphs as we approach the singularity.

Joshua Cruz

PhD Thesis

Examples of the Local L^2-cohomology of Algebraic Varieties



The relationship between analysis and topology in the case of manifolds is given by de Rham cohomology. We would like to do something similar for these singular spaces, but we instead use L^2-cohomology, which is similar to de Rham cohomology except we require L^2 growth conditions on our forms as we approach the singularity. Calculating L^2-cohomology can be difficult in general, because common techniques like the Mayer-Vietoris sequence are not applicable in this context. My thesis introduces new tools for calculating L^2-cohomology for singularities like the ones described above. These are applied to calculate the L^2-cohomology of several examples of both real and complex algebraic varieties, including the A_n hypersurface singularities in arbitrary dimension.

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