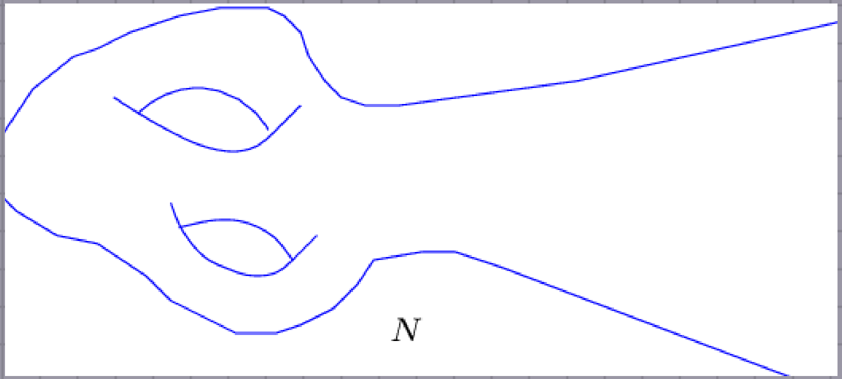
Geometric flows, such as the Ricci flow, are partial differential equations on smooth manifolds which describe the time evolution of some geometric structure on the manifold, such as a Riemannian metric. Many geometric flows are heuristically like the heat equation in that they can be thought of as deforming an initial geometric structure to one that is nicer in some sense. The nonlinearity of many geometric flows frequently leads to the formation of singularities in finite time. To understand these singularities, we study special, self-similar solutions called solitons, with the expectation that developing singularities of the flow are modeled on these soliton solutions. In the setting of G2 geometry, which is specific to seven dimensions, a natural geometric flow to consider the is (closed) G2 Laplacian flow. Critical points of this flow correspond to torsion-free G2 structures, which satisfy a system of nonlinear partial differential equations. We are interested in such G2 structures since they give rise to Riemannian metrics with exceptional holonomy.

Anuk Dayaprema

Senior Thesis

Solitons for the closed G2 Laplacian flow

Since the soliton equation is a nonlinear system of PDE, to get a feel for concrete solutions we consider a dimensional reduction of the equation by studying cohomogeneity-one solitons. We consider the cohomogeneity-one setting where the group G of isometries is either SU(3) or Sp(2). In these regimes, we show that the soliton equation is equivalent to a (nonlinear) first-order system of real-analytic ODEs. We then describe the space of closed G-invariant cones, identifying it with a smooth surface/curve in R3.



We then investigate the uniqueness of a complete shrinking Sp(2)-invariant soliton which possesses an asymptotically conical geometry, and conjecture that it is rigid to first order. We finally discuss ways to study the general closed Laplacian soliton equation, reviewing previous results by others in related directions, with an eye towards asymptotically conical shrinking solitons in particular.

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