Blythe Davis

The Spherical Manifold Realization Problem

A 3-manifold is a 3-dimensional space that locally resembles Euclidean space. One example of a 3-manifold is the 3-sphere. A knot in the 3-sphere is an embedding of the circle into the 3-sphere, and links are embeddings of multiple circles. One way to transform a 3-manifold into another 3-manifold is through Dehn surgery on a knot or link, a process in which a tube, or a solid torus, in the shape of the knot is removed from the original 3-manifold and replaced with a solid torus in a way specified by a curve on boundary of the original torus that is given by a rational number called the surgery coefficient. In this project, we consider which spherical manifolds (a specific family of 3-manifolds) can be obtained by surgery on knots in the 3-sphere with integral surgery coefficients.

The Lickorish-Wallace theorem states that any closed, orientable, connected 3-manifold can be obtained by integral Dehn surgery on a link in the 3-sphere The spherical manifold realization problem asks which spherical manifolds (i.e., those with finite fundamental groups) can be obtained through integral surgery on a knot in the 3-sphere. The problem has previously been solved by Greene and Ballinger et al. for lens space and prism manifolds, respectively. In this project, we determine which of the remaining three types of spherical manifolds (tetrahedral, octahedral, and icosahedral) can be obtained by positive integral surgery on a knot in the 3-sphere.