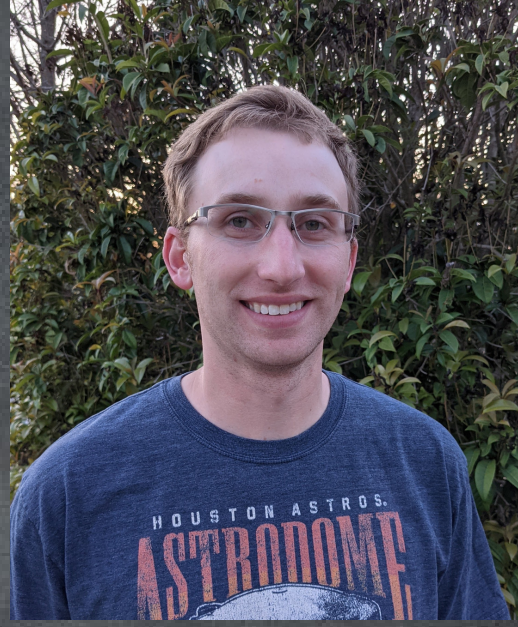
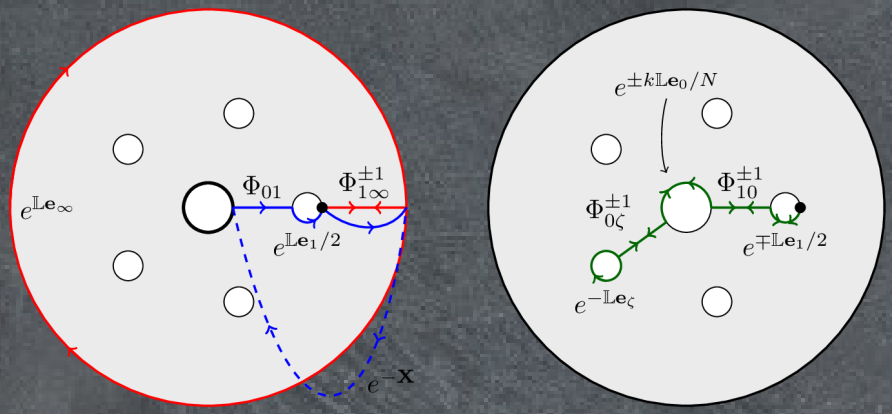
A period is a complex number which can be expressed as an integral of an algebraic function over an algebraic domain. Examples include the algebraic numbers, π, and integer values of the Riemann zeta function. Periods may also be interpreted as matrix entries of the comparison isomorphism between the de Rham and Betti cohomology of an algebraic variety. From this perspective, periods arise from the more abstract notion of motives.

Cyclotomic and Elliptic Polylogarithms and Motivic Extensions of Q by Q(m)



Eric Hopper

PhD Thesis



Deligne and Goncharov showed the category MTM(N) of mixed Tate motives unramified over the integers with the Nth roots of unity adjoined contains the values of multiple polylogarithms at Nth roots of unity, which we call N-cyclotomic multiple zeta values (MZVs), as periods. When N = 1, relations between these periods are known to correspond to cusp forms of SL(2,Z). The goal of my thesis is to begin to understand the analogous relationship between N-cyclotomic MZVs and cusp forms of level N when N > 1. To do this, we use the level N KZB connection to show the Lie algebra of the unipotent fundamental group of the first order smoothing of the nodal cubic with a cyclic subgroup of order N removed is an object of MTM(N). We then compute the resulting action of the fundamental Lie algebra of MTM(N) on this object explicitly in depth 1. The relations between the generators of this fundamental Lie algebra in depth 2 are dual to relations between N-cyclotomic MZVs and expected to correspond to cusp forms of level N.

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