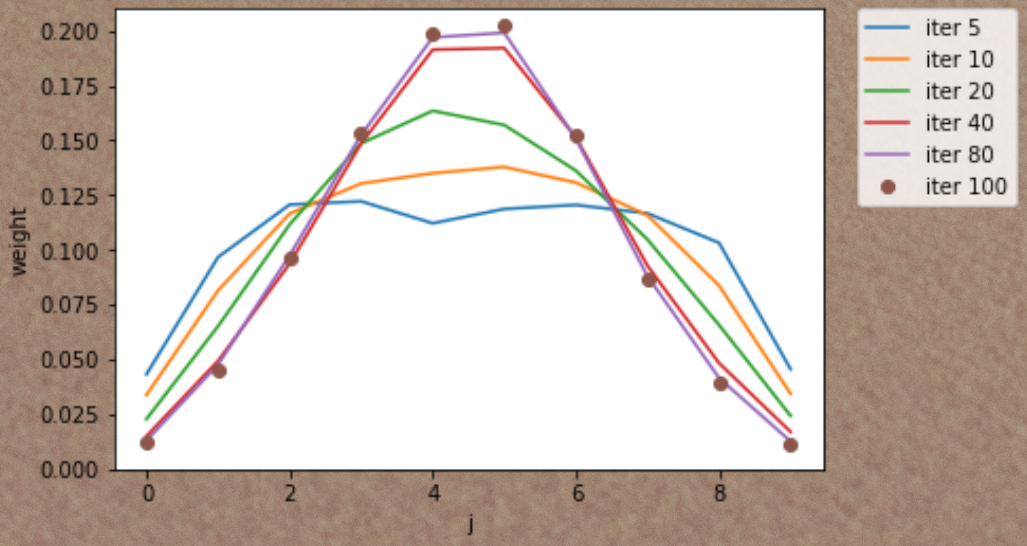
Markov Chain Monte Carlo (MCMC) methods seek to draw samples from a probability distribution by simulating a particle that eventually settles into the distribution. In many real-life situations, the space a particle explores is divided into sections, so that it is hard for it to get from one to another. Stratified MCMC algorithms approach these cases by dividing the space up into separate parts, or "strata," and simulating particles in each one. This gives us a sample from each stratum. Particles are not allowed to leave their own strata, but the algorithm keeps track of when and how they try to leave. This information is then used to patch together the samples, and get an idea of what the entire distribution looks like. In my thesis, I examine one such algorithm is based on estimating how particles entering each strata are distributed.

Stratified MCMC Sampling for Non-Reversible Dynamics

Gabe Earle

PhD Thesis

The Injection Measure Method begins with a sample space and a partition of subsets of the space, called strata. Starting with some initial distributions on the strata, and a set of initial weights for them, it then simulates the path a particle following some Markov chain takes from each initial distribution until exiting its stratum. The exit points are then used to build new distributions, called the injection measures, on each stratum, and new weights. We consider two versions: the basic version computes the new weights based on the old ones and how the particles move between strata, and the eigenvector version creates a transition matrix on the set of strata based on how particles move between them, then solves for its principle eigenvector for the new weights. The injection measures can then be patched together at each iteration, with the corresponding weights, to form a total distribution on the whole space. I use coupling arguments to prove that, in total variation and under certain conditions, the resulting distributions converge geometrically to the invariant measure of the original Markov chain. I also prove bounds on the rate of convergence and discuss their interpretation, as well as demonstrate an example of the method in practice via numerical simulation.



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