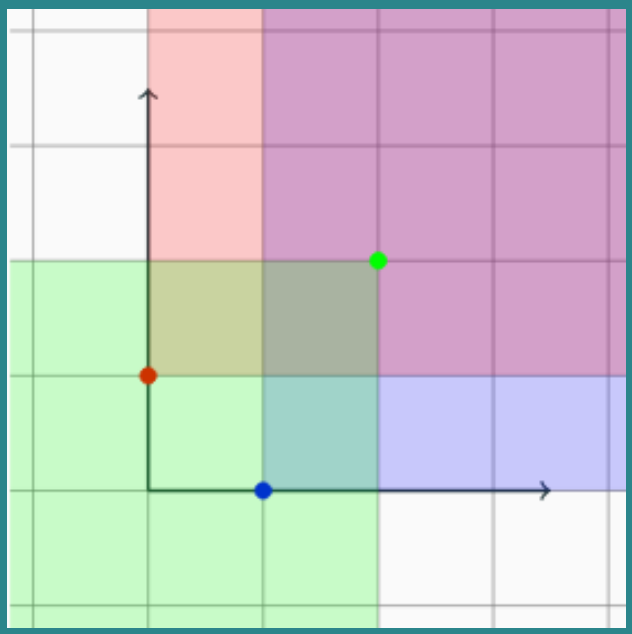
Topological data analysis is an area of data science which seeks to find topological (or shape) features in data. It has seen increasing usage in recent years, finding applications in settings ranging from analyzing brain artery trees to visualizing the geometric structures of music. The flagship technique in topological data analysis is known as single-parameter persistent homology, which helps detect shape features in data which persist over a wide range of values for a single parameter. Many in the field have sought to extend this technique to work for two or more parameters, but the mathematical theory for doing so is currently relatively sparse. We seek to provide the beginnings of a mathematical framework toward extending a crucial component of the single-parameter theory, namely the decomposition of modules, to the case with multiple parameters. In particular, we define an algebraic data structure called the QR code and proving that it can encode all the relevant information of a module, and begin to reframe the problem of decomposing modules in terms of our new data structure.

Joey Li

Senior Thesis

Algebraic Data Structures for Decomposing Multipersistence Modules

Single-parameter persistent homology techniques in topological data analysis have seen increasing usage in recent years. These techniques have found particular success because of the existence of a complete, discrete, efficiently computable invariant to describe persistence modules in the single-parameter case: the barcode. Attempts to develop an equally robust theory of multiparameter persistent homology, however, have been slow to progress because there is no natural multiparameter analogue to the barcode. Relatively little is known about the structure of decompositions of multiparameter persistence (multipersistence) modules or how to classify their indecomposables. In fact, even for the problem of computing decompositions, there currently is no generalization to multiple parameters of the decomposition algorithm from single-parameter persistent homology. We define a new algebraic data structure, the QR code, which was first proposed in an earlier preprint by Professor Ezra Miller, but was formulated somewhat erroneously. Additionally, we prove a theorem stating that the QR code recovers all the information of the module it encodes. We suggest that this new data structure, which seeks to encode a module using births and deaths rather than births and relations, may be the correct language in which to solve the problem of decomposing arbitrary finitely generated multipersistence modules.



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