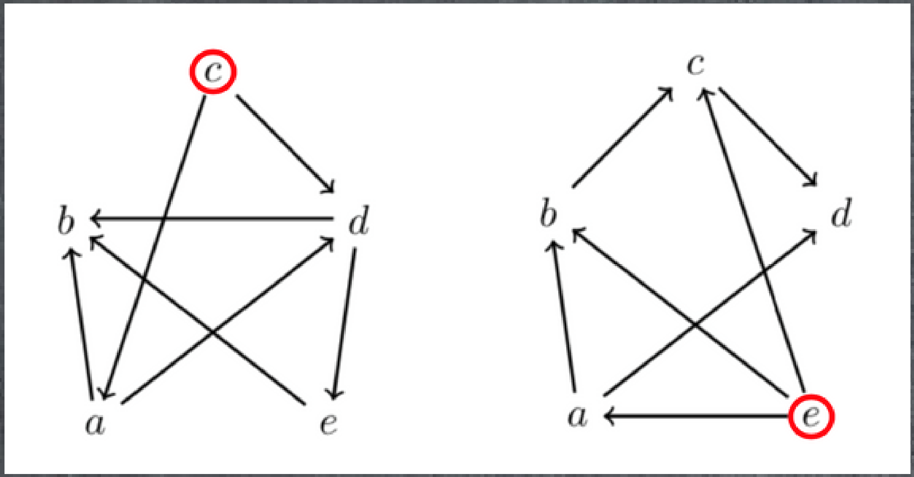
When most people think of an election, they are picturing the plurality voting method: each person votes for their favorite option and the winner is the option with the most votes. This works perfectly when choosing between two options, but problems can arise when there are three or more options. For example, in an election with three candidates A, B, and C, it is possible for A to be preferred over B by a majority of the voters and over C by a majority of the voters but still lose an election by plurality. How is this possible? Say 40% of the electorate prefers B over A over C. Say 30% prefers C over A over B, and say 30% prefers A over B over C. Here, A is called a Condorcet candidate because it wins every head to head matchup against any other candidate. Many people think that if a Condorcet candidate exists, then they should always win the election. This motivates the Condorcet Criterion which is satisfied by a voting method if that voting method always elects Condorcet candidates when they exist. The plurality method does not satisfy this criterion, but many other methods which take voters' full preferences into account do. Ranked Pairs is a voting method that satisfies the Condorcet Criterion along with several other desirable criteria. The goal of this thesis is to determine whether or not there is some set of criteria that Ranked Pairs is unique in satisfying.

Jonathan Michala

Senior Thesis

Uniqueness of Ranked Pairs



The method itself is quite simple and natural; it takes each voter's full preferences and creates a list of pairwise matchups between all candidates, ordered by magnitude of victory. Then, it implements each pairwise victory in that order, unless it would cause a cycle of candidates beating each other. With no cycles, the method outputs a complete ordering of the candidates with the top candidate as the winner. The four main criteria satisfied by Ranked Pairs are Monotonicity, Local Irrelevance of Independent Alternatives, Independence of Clones, and Independence of Smith-Dominated Alternatives (which implies the Condorcet Criterion). In this thesis, we show that in a special case restricted to 3 candidates, Ranked Pairs is not unique in satisfying these criteria, and we make a conjecture that this extends to the general case with any number of candidates. We also introduce a stronger notion of Monotonicity that gives more progress in the proof of uniqueness, and we conjecture that Ranked pairs is unique with this altered set of criteria. Other findings of this thesis include the surprising failure of Ranked Pairs to satisfy a certain notion of Monotonicity, a new conception of the Smith Set involving tiers of candidates, and a proof that a voting method which satisfies the Majority Criterion and Local Independence of Irrelevant Alternatives also must satisfy Independence of Smith-Dominated Alternatives.

Duke UNIVERSITY MATH