The focus of my project is computing explicit parametrizations describing a certain type of 2-dimensional surface sitting in 4-dimensional space. The approach utilizes objects studied in differential geometry to produce a system of differential equations that when solved yield an expression for our desired surface. In full generality, this is very difficult, so I introduce additional properties I want the surface to satisfy, which translate into additional tools that can be used to simplify system of equations. More specifically, I am solving for embeddings of Lagrangian surfaces which are completely determined by Maurer-Cartan forms satisfying Cartan's structure equations due to a theorem by Bonnet. Differentiating the structure equations in two ways produces a system of PDEs, which can be simplified into a system of ODEs if we force the surface to also be homogeneous or contain 1-parameter symmetry group. Once solutions are found, software like Maple can be used to plot graphs such as the one below.

Junmo Ryang

Senior Thesis

Embedding Lagrangian Surfaces

The motivation for my project was to understand the conditions under which a surface sitting in Euclidean 3-space could be isometrically embedded as a Lagrangian surface in Euclidean 4-space. In practice, I have mostly focused on computing explicit parametrizations for some special cases of Lagrangian surfaces by solving a Bonnet problem. The primary method employed uses Cartan's theory of moving frames to produce a system of differential equations: by differentiating the Maurer-Cartan forms associated with a frame field adapted to the surface, we arrive at a set of necessary and sufficient conditions for the existence of a Lagrangian surface, at least locally. Focusing on the more tractable cases, such as when the embedded surface is homogeneous or has a 1-parameter symmetry group, the system reduces to a system of ODE’s. Each solution to this system produces Maurer-Cartan forms corresponds to a local embedding of a surface, which can often times be found by quadrature. Enforcing certain geometric properties such as constant Gauss curvature or zero mean curvature often further simplifies the system, while conversely some free parameters in the solutions maybe amenable to some geometric interpretation. Maple was a key tool for both computation and visualization.

Duke UNIVERSITY MATH

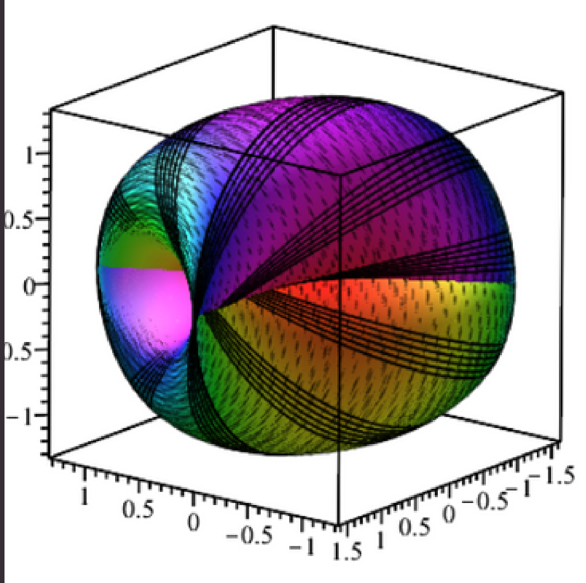

FIGURE 6. $u = 0, v = 1$