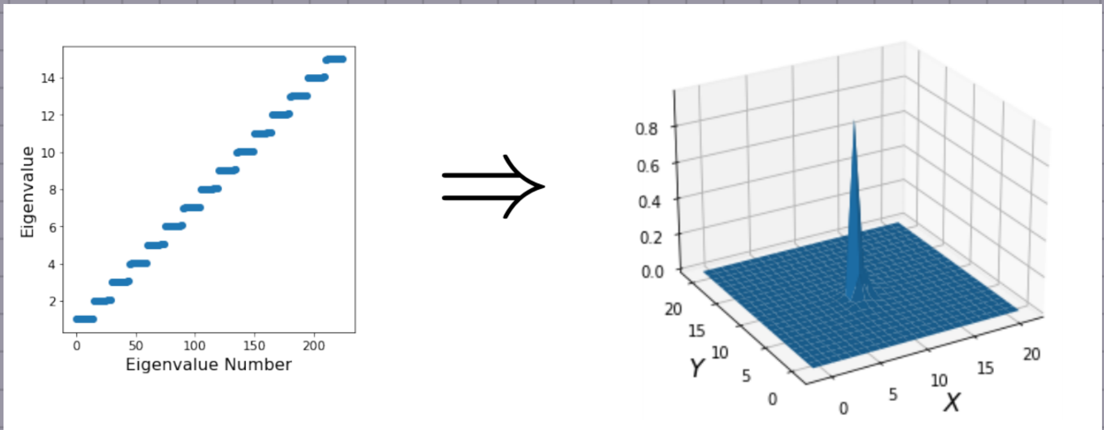
Ice melts to make water, water boils to make steam. Transitions between the different phases of matter (solid, liquid, gas, and plasma) make up an important part of our daily lives. Oftentimes we can understand what phase a material is in by knowing its temperature and pressure. Importantly, parameters like temperature and pressure can take any value in a reasonable range. For example, it can be 73 degrees outside or 85 degrees or 73.2 degrees. In the early 1980s, it was realized that there are also parameters which predict different phases of matter that can only take on discrete values. That is, the underlying parameter can be 0, 1, 2, or 3, but never .5 or 1.25. This new type of parameter is deeply connected with a field of mathematics known as topology. Hence this new class of parameters are referred to as "topological order parameters.'' In the same way that steam and ice have different properties, materials with different topological order parameters also have different properties. Recently, it has been found that materials with a non-zero topological order parameter can exhibit interesting behaviors, such as the ability to conduct electricity with essentially zero resistance.

Kevin Stubbs

PhD Thesis

On Exponentially Localized Wannier Functions in Non-Periodic Insulators

In my dissertation, I consider insulating electron systems in two dimension whose dynamics are described by an effective single-particle Hamiltonian. For such systems, the subspace of physically relevant states is defined by an orthogonal projection called the Fermi projection. In periodic systems, we can determine whether the system has non-zero topological order parameter by calculating a topological invariant associated with the Fermi projector known as the Chern number. Recently, it has been shown for a wide class of periodic systems that the Chern number is zero if and only if the Fermi projection admits a well localized basis. The main contribution of my dissertation is to propose and prove the correctness of an algorithm for calculating well localized bases for the Fermi projection in both periodic and non-periodic insulators. As a consequence of our proof, we establish sufficient conditions for when a well localized basis for the Fermi projection exists in non-periodic systems. These results open the door for extending the theory of topological order parameters to fully non-periodic systems.



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