MATH 490: Numerical Algebraic Geometry, Spring 2022

TTh 3:30 - 4:45 pm ET in Physics 259

Instructor: Dr. Margaret (Maggie) Regan (mregan@math.duke.edu, www.margaretregan.com)

Contact Method: Please contact me by email with questions, comments, concerns, or requests for meetings or help. I will respond within 24 hours.

Office Hours: TBD

Books: No required text.

Optional Reading: *Numerical Analysis: Mathematics of Scientific Computing* (3rd Ed.) Kincaid-Cheney, 2002.
Numerical Linear Algebra, L.N. Trefethen and D. Bau, III, SIAM, 1997.
Numerically Solving Polynomial Systems with Bertini, Bates, Hauenstein, Sommese, and Wampler, 2013.

Description: Systems of equations arise in many areas of science and engineering along with a wide variety of numerical solving techniques. When the equations are polynomial, algebraic geometry can be combined with numerical methods based on homotopy continuation to develop rigorous numerical solving algorithms. This course will introduce students to such algorithms, collectively called *numerical algebraic geometry*, from a computational viewpoint. The material will begin with a solid introduction to numerical analysis for solving linear systems, collectively called *numerical linear algebra*, move through topics such as polynomial interpolation and ordinary differential equations, and end with a discussion on local solving methods such as Newton's method and global solving methods using homotopy continuation. At the end of the semester, students will complete a project and give a presentation regarding solving nonlinear systems of equations in an application of their choice. *Prerequisites*: MATH 216/218/221, or equivalent.

Attendance: Students are expected to *actively* attend every class lecture in person. In the online learning environment, "attendance" is measured by your *presence* on the site and your *contributions* to the site. If a student is in quarantine or has a major time zone misalignment, contact me and we will discuss attendance and class participation accommodations. Excessive unexplained absence may be reported to the dean's office of the college.

Electronics: Please respect your fellow students and prevent your electronics from disrupting class. If entering via a Zoom meeting, make sure to mute your audio. I encourage you to attend any Zoom meetings with your camera on as it helps us all communicate better.

Topics: As time permits, we will cover the following topics:

- Numerically solving linear and nonlinear equations
- Polynomial interpolation
- Numerically solving ordinary differential equations (ODEs)
- Global and local solving methods
- Homotopy continuation

Objectives: As time permits, the students will achieve the following objectives:

1. Students will be able to effectively use definitions, examples, theory, and algorithms from the topics outlined above.

2. Students will be able to recognize and write valid, rigorous proofs.

3. Students will be able to effectively code numerical algorithms.

4. Students will be able to discuss mathematics, including: presenting solutions via zoom, generating examples for illustration as appropriate, seeking and finding holes in proposed proofs, code algorithms for numerically solving problems.

Software and Technology: Implementation of algorithms is essential for this course. Students are free to choose the language they will use, but in-class demonstrations will mainly use MATLAB (available for free to students from Duke OIT). Duke OIT can also assist with any Zoom or Sakai issues. You can also access the self-service help documentation for Zoom here and for Sakai here.

Collaboration and Honor Code: Students are permitted and encouraged to work together when doing homework, but copying work is not allowed. Include the names of any collaborators at the top of your homework submission. Examinations and homework are conducted under the Duke Community Standard.

Absences and Makeup Work: Students are expected to arrive on time, stay the entire class, and contribute to the class discussion and group work. Excused absences and makeup exams will be handled according to University policy. Please notify the instructor in writing (email message is acceptable) prior to the date of absence when this is feasible. In cases where advance notification is not feasible (e.g., accident or emergency), the student should notify me as soon as possible.

Grading: The grading scheme will be the following:

Homework & Participation	40%
Midterms 1 & 2	2 @ 15% each
Final Project	30%

Each component of the above grade is calculated based on percentages.
The final grades will follow the breakdown below:

A+ ≥ 97%	B+ ≥ 87%	C+ ≥ 77%	D+ ≥ 67%	F < 60%
A ≥ 93%	B ≥ 83%	C ≥ 73%	D ≥ 63%	
A- ≥ 90%	B- ≥ 80%	C- ≥ 70%	D- ≥ 60%	

Homework will be submitted using Gradescope and is due every week. *Late homework will not be accepted.* Homework will be graded via the following 3 point scale:

3 points = fully correct, all work present
2 points = mostly correct, possibly missing work
1 points = mostly incorrect, missing most work
0 points = no solution

Class participation and office hour attendance can be used to boost homework grade.
The final project involves a written report and giving a presentation regarding
solving nonlinear systems of equations in an application of the student's choice.

Exam Dates
Midterm 1: Thursday, February 17 in Physics 259
Midterm 2: Thursday, March 17 in Physics 259
Final presentations: TBD — during registrar schedule final exam time
** Midterm exams will involve both an in-class and take-home portion.

Appeal: All appeals related to homework and exam grades must be submitted within 1 week after they are graded. To appeal, the student must submit to me the following: homework question or exam, written or typed note explaining which question(s) is/are being appealed, and the basis for the appeal (e.g., the question was mistakenly marked incorrectly, etc.). I will review each appeal and make appropriate changes.

Disabilities: Reasonable accommodations will be made for students who are registered with the Student Disability Access Office. Such students should speak with me as soon as possible.

Inclusivity: Duke University's Office for Institutional Equity provides resources, events, and information about current initiatives at Duke to support equality for all members of the Duke community. I hope that you will communicate with me if you experience anything in this course that does not support an inclusive environment, and you can also report any incidents you may witness or experience on campus to the Office for Institutional Equity.

Mental Health and Wellness: If your mental health concerns and/or stressful events negatively affect your daily emotional state, academic performance, or ability to participate in your daily activities, many resources are available to you, including ones listed below. Duke encourages all students to access these resources, particularly as we navigate the transition and emotions associated with this time. Duke Student Government has worked with DukeReach and student advocates to create the "Two-Click Support" Form, and DukeReach has expanded its drop in hours as well. Other resources available are the following:

- **DukeReach.** Provides comprehensive outreach services to identify and support students in managing all aspects of wellbeing. Learn more here.
- **Counseling and Psychological Services (CAPS).** CAPS services include individual group, and couples counseling services, health coaching, psychiatric services, and workshops and discussions. (919) 660-1000
- **Blue Devils Care.** A convenient and cost-effective way for Duke students to receive 24/7 mental health support through TalkNow. Learn more here.

In addition, managing daily stress and self-care are also important to well-being. Duke offers several resources for students to both seek assistance on coursework and improve overall wellness, some of which are listed below. Learn more here.

- **The Academic Resource Center (ARC)**. (919) 684-5917, theARC@duke.edu
- DuWell. (919) 681-8421, duwell@studentaffairs.duke.edu
- WellTrack.