Matt Beckett

PhD Thesis

My area of research is in Gauge Theory, in the intersection of geometry, topology, analysis, and physics. Gauge Theory was developed as a generalization of electromagnetic field theory that became fundamental to our understanding of physics at small scales. Mathematically, we are interested in studying the geometry of spaces of solutions of gauge-theoretic equations. One important tool in this pursuit is the Nahm Transform, which gives a convenient description of Yang--Mills fields in terms of a dual geometric construction. Among other projects, in my dissertation, I constructed a Nahm Transform for solutions satisfying certain symmetries, and as such I was able to provide a description of rotationally symmetric Yang--Mills fields.

Equivariant Nahm Transforms and Minimal Yang-Mills Connections

Instantons, which are a special type of minimal Yang--Mills field, are objects of particular interest in Gauge Theory. The Nahm Transform is often described as a Fourier Transform for instantons, and is an important tool in studying the geometry of spaces of instantons. Given an instanton over a flat space, one can construct a connection on some dual space, and this dual connection itself often satisfies some instanton-like equation. The ADHM construction, which describes all instantons on Euclidean space, can be viewed as a special case of the Nahm Transform. By considering an equivariant formulation of the Nahm Transform,I constructed an ADHM construction specifically for rotationally-invariant instantons, which themselves are in correspondence with singular monopoles. As such I was able to rovide explicit descriptions of moduli spaces of singular monopoles. By instead considering a crystallographic action on Euclidean space, I also constructed a Nahm Transform for instantons on a flat manifold with nonabelian fundamental group. In my dissertation, I also generalized some results showing that minimal-energy Yang--Mills fields are instantons.

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