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Senior Thesis

Khovanov Homology and Knot Concordance

Khovanov homology (Kh) is a link homology theory, which is to say that it accepts as knot/input a link and outputs a bi-graded group Kh(L). Let S be a surface such that at its two boundaries are links L and L'. Such a surface is called a cobordism. Then it is well-known that there exists a well-defined map, called Kh(S), from the Kh(L) group to the Kh(L') group. The Kh(S) map is unchanged if S is changed by an isotopy in 4-space. This is to say that Khovanov homology is functorial up to isotopy of cobordisms. This project is about showing that when the boundary of S is split, then the Kh(S) map "forgets" the linking information between distinct components of S. The attached image shows this in a picture. The left and right hand sides induce identical maps on Khovanov homology, even though they represent different surfaces. Intuitively speaking, what this tells us is that if the boundary of S is split, we can separate (split apart) the individual components of S, without altering the Kh(S) map. This has applications in the study of special cobordisms known as strongly homotopy ribbon concordances.

The functoriality of Khovanov homology tells us that altering a cobordism by an ambient isotopy in 4 dimensions preserves the induced Khovanov map. This project shows that if the cobordism goes between split links, then we can separate each component by a copy of S^2 x [0,1], without altering the induced Khovanov map. This tells us something interesting about the Khovanov functor by pointing to a weakness that forgets the linking information between distinct components. This "splitting statement" allows us to extend Levine-Zemke's result to strongly homotopy ribbon concordances: If C is a strongly homotopy ribbon concordance, then it induces an injective map on Khovanov homology. The "splitting statement" also allows us to fully characterise the Khovanov-Jacobsson number of closed cobordisms.

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