

Instructions: This self-assessment is designed for Duke students planning to enroll in Calculus I who are unsure whether Math 105L or Math 111L is the best fit.

In addition to offering this Precalculus Self-Assessment to help you place yourself before registering, all Calculus I students will take a Calculus I Placement Test covering precalculus material on the first day of class. Students must score above threshold on that test to take Math 111L.

For the most accurate guidance from this self-assessment, we recommend that you

- Work through the 11 questions below within a single 2-hour sitting;
- Write your solutions by hand on a separate sheet of paper, showing all work so that your reasoning and process are clear;
- Do not make use of any outside resources, including textbooks, calculators, the internet, and other people.

When finished or the 2 hours end, click on the link to the answers and score your work.

1. Find the slope-intercept equations of the following 2 lines.

- (a) The line which passes through the point $(-5, 1)$ and is parallel to the line through the points $(3, 7)$ and $(1, -1)$.
- (b) The line which passes through the point $(2, -3)$ and is perpendicular to the line given by the equation $3x - 4y = 10$.

2. If $f(x) = 2x^2 - 3x$ and $g(x) = x + 2$, evaluate each of the following 6 expressions, simplifying your final answer as much as possible.

- (a) $f(x) + 2g(x)$
- (b) $f(x)g(x)$
- (c) $f(g(x))$
- (d) $\frac{f(1)}{g(1)}$
- (e) $\frac{f(1+h) - f(1)}{h}$
- (f) $\frac{g(x+h) - g(x)}{h}$

3. Determine whether or not each of the following 3 functions is invertible. If the function is invertible, find its inverse. If the function is not invertible, explain why not.

- (a) $f(x) = \frac{x}{x+2}$
- (b) $g(x) = (x-1)^2 + 3$
- (c) $h(x) = \ln(3x+1) - 5$

4. Simplify each of the following expressions fully, so that x and y each appear once.

- (a) $\frac{x^3y^5x^{-2}}{x^{-2}y^2}$
- (b) $\frac{\sqrt[4]{16x^6y^{14}}}{\sqrt[5]{x^2y^5}}$

5. Let $f(x) = \log_{10}(x) = \log(x)$.

- State the domain and range of the function $f(x)$.
- Find $f^{-1}(x)$.
- Find the exact value of $f(\sqrt{1000}) + f\left(\frac{1}{10}\right)$.
- Find the exact value of $f(30^2) - f(90)$.

6. Consider the function $f(x) = \frac{(9x^2 - 4)(2x + 1)}{x^3 + 2x^2 + 5x + 10}$.

- What is the domain of $f(x)$?
- What are the roots (zeroes), if any, of $f(x)$?
- What are the vertical and horizontal asymptotes, if any, of $f(x)$?

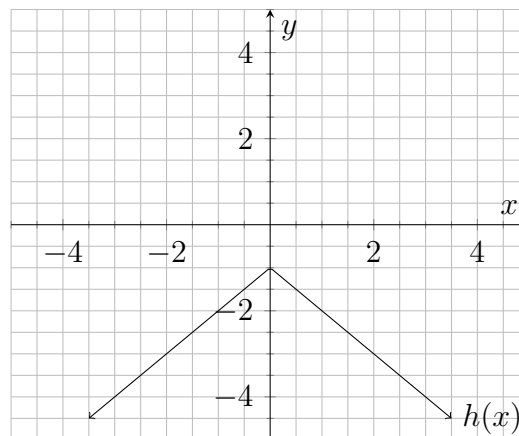
7. For each of the 3 functions below,

- Carefully sketch the graph, identifying and labeling any intercepts.
- Give the domain and range.

(a) $f(x) = (x + 3)^2(x - 1)^2(x - 2)$

(b) $g(x) = 2\sin\left(-\frac{1}{2}x\right)$

(c) The function $-h(2x) - 3$ given the graph at right of $h(x)$.



8. At a certain grocery store, apples cost

- \$3.00 per pound if you buy 2 pounds or fewer,
- \$2.50 per pound if you buy more than 2 pounds but less than 8 pounds, and
- \$2 per pound, plus a flat fee of \$1.50 regardless of total weight purchased (this covers the cost of the carrying crate), if you buy 8 pounds or more.

- How much does it cost to purchase 4 pounds of apples?
- How much does it cost to purchase 8 pounds of apples?
- Write down the piecewise function $C(a)$ giving the cost, in dollars (\$), of purchasing a pounds of apples.

9. Find all solutions, if any, of each of the following 11 equations, inequalities, and systems. In (d), (e), (f), only find solutions on the specified domain, $0 \leq x \leq 2\pi$.

(a) $\sqrt{5x+2} - 4 = 6$

(b) $2^{3x-1} = 5^{x-2}$

(c) $\log(x+2) - \log(x) = \log(3)$

(d) $4 \cos(3x) + 1 = 5$
over the interval $0 \leq x < 2\pi$

(e) $2 \tan^2 x + 2 \tan x = 0$
over the interval $0 \leq x < 2\pi$

(f) $\sin^2(x) + \cos^2(x) = 4x - 6$
over the interval $0 \leq x < 2\pi$

(g) $|3x - 2| \geq 1$

(h) $\frac{(x+1)(x-2)}{x-3} \leq 1$

(i) $\frac{1}{x+2} > \frac{3}{x}$

(j) $\begin{cases} y = x^2 + 2x + 3 \\ y = 5x + 7 \end{cases}$

(k) $\begin{cases} 2x - y + z = 1 \\ x - y + 2z = 3 \\ x - y + z = 1 \end{cases}$

10. Let t be the measure, in radians, of an angle in the first quadrant, as pictured below. Evaluate the following expressions in terms of a and t .

(a) The measure of t in degrees

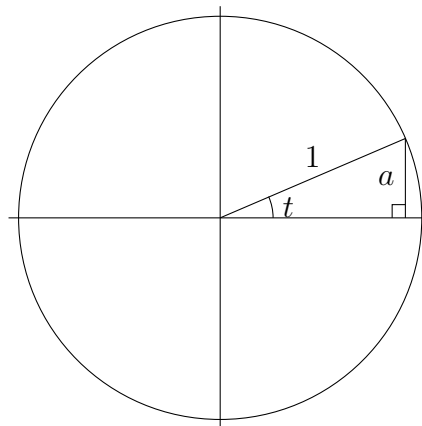
(b) $\sin(t + 2\pi)$

(c) $\cos\left(\frac{\pi}{2} - t\right)$

(d) $\sin(2\pi - t)$

(e) $\arcsin(a) = \sin^{-1}(a)$

(f) $\arccos(-a) = \cos^{-1}(-a)$



11. A 6-foot tall post stands between a wall and a light on the ground, as pictured at right. The post is 8 feet away from the light and 12 feet away from the wall. The wall forms a right angle with the ground.

(a) What is the distance from the top of the post to the light? (This is labeled a in the figure.)

(b) What is the height of the shadow cast by the post on the wall? (This is labeled b .)

(c) What is the distance from the top of the post to the top of the shadow? (This is labeled c .)

(d) If θ is the measure of the angle formed by the light and the ground, determine the exact value of each of the following:

i. $\sin \theta$

iii. $\tan \theta$

v. $\sec \theta$

ii. $\cos \theta$

iv. $\cot \theta$

vi. $\csc \theta$

