

Instructions: Please self-score your work out of 76 points using the answers and point values below. We recommend that

- Students who score a total above 53 enroll in Math 111L Calculus I;
- Students who scored a total below 53 enroll in Math 105L Calculus I to strengthen their understanding of precalculus material.

Recall this is a self-assessment only: all Calculus I students (both those originally enrolled in Math 105L and those in Math 111L) will take a Calculus I Placement Test on precalculus material on the first day of class. Regardless of results on this self-assessment, students must score above threshold on that test to take Math 111L.

Topics for each question are provided below with the answers if you are interested in reviewing material on your own before the semester begins.

To discuss Calculus I placement, we encourage you to reach out the Math 105L and Math 111L course coordinators, Prof. Rann Bar-On and Prof. Shira Viel, respectively, at [rann.baron@duke.edu](mailto:rann.baron@duke.edu) and [shira.viel@duke.edu](mailto:shira.viel@duke.edu).

- (a)  $y = 4x + 21$     (b)  $y = -\frac{4}{3}x - \frac{1}{3}$

  - 4 points: 2 points per part- 1 for slope, 1 for intercept
  - Topic: Lines, including parallel and perpendicular slopes.
- (a)  $2x^2 - x + 4$ ,    (b)  $2x^3 + x^2 - 6x$     (c)  $2x^2 + 5x + 2$ ,    (d)  $-\frac{1}{3}$ ,    (e)  $2h + 1$ ,    (f) 1

  - 6 points: 1 point per part
  - Topic: Functions, including function composition and difference quotients
- (a)  $f^{-1}(x) = -\frac{2x}{x-1}$ ,    (b) The function  $g(x)$  is not invertible since it is not one-to-one; for example,  $g(2) = 4 = g(0)$ ,    (c)  $h^{-1}(x) = \frac{1}{3}e^{x+5} - \frac{1}{3}$

  - 6 points: 2 points per part (1 point for invertible/not, 1 point for inverse/explanation)
  - Topic: Inverse functions
- (a)  $x^3y^3$     (b)  $2x^{11/10}y^{5/2}$

  - 2 points: 1 point per part
  - Topic: Properties of Exponents
- (a) Domain:  $x > 0$ , Range: All real numbers,    (b)  $f^{-1}(x) = 10^x$ ,    (c)  $\frac{1}{2}$ ,    (d) 1

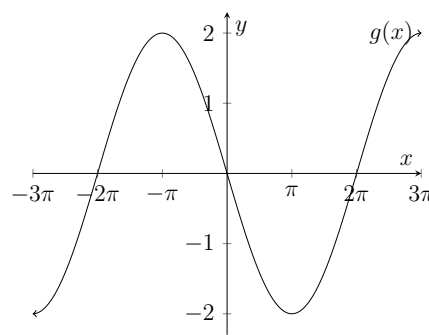
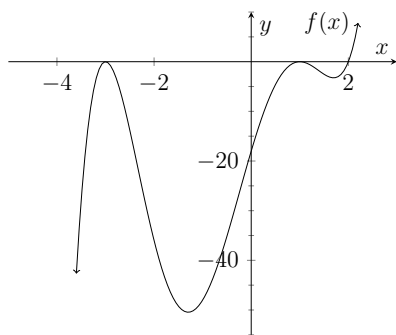
  - 4 points: 1 point per part (1/2 pt each for domain and range in (a))
  - Topic: Properties of Logarithms

6. (a) Domain:  $x \neq -2$ , (b)  $x = -\frac{2}{3}, x = \frac{2}{3}, x = -\frac{1}{2}$ , (c) VA  $x = -2$ , HA  $y = 18$ .

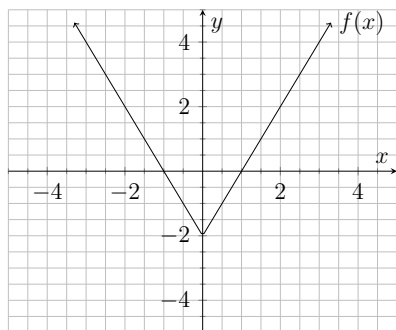
- 6 points: 1 point (a); 3 points (b)- 1 per root; 2 points (c)- 1 per asymptote
- Topics: Factoring, Rational Functions

7. See graphs, intercepts, domains (D), and ranges (R) below.

(a) D: All real numbers, R: All real numbers (b) D: All real numbers, R:  $-2 \leq x \leq 2$   
 x-intercept(s):  $(-3, 0), (1, 0), (-2, 0)$  x-intercept(s):  $\dots -4\pi, -2\pi, 0, 2\pi, 4\pi, \dots$   
 y-intercept:  $(0, -18)$  y-intercept:  $(0, 0)$



(c) D: All reals, R:  $x \geq -2$   
 x-intercept(s):  $(-1, 0), (1, 0)$   
 y-intercept:  $(0, -4)$



- 9 points: 3 points per part (1 for graph, 1 for intercepts, 1 for domain and range)
- Topics: Graphing polynomial and trigonometric functions, graph transformations

8. (a) \$10, (b) \$17.50, (c)  $C(a) = \begin{cases} 3a & \text{if } 0 \leq a \leq 2 \\ 2.5a & \text{if } 2 < a < 8 \\ 2a + 1.5 & \text{if } a \geq 8 \end{cases}$

- 5 points: 1 point (a), 1 point (b), 3 points (c)- 1 point per piece.
- Topic: Piecewise functions

9. (a)  $x = \frac{98}{5}$  (f)  $x = \frac{7}{4}$   
 (b)  $x = \frac{\ln 2 - 2 \ln 5}{3 \ln 2 - \ln 5}$  (g)  $x \geq 1$  or  $x \leq -\frac{1}{3}$   
 (c)  $x = 1$  (h)  $-1 \leq x \leq 3$   
 (d)  $x = 0, x = \frac{2\pi}{3},$  or  $x = \frac{4\pi}{3}$  (i)  $x < -3$  or  $-2 < x < 0$   
 (e)  $x = 0, x = \frac{3\pi}{4}, x = \pi,$  or  $x = \frac{7\pi}{4}$  (j)  $(x, y) = (4, 27)$  or  $(x, y) = (-1, 2)$   
 (k)  $(x, y, z) = (0, 1, 2)$

- 22 points: 2 points per part (2/3 pt per answer component in (d), 1/2 pt per component in (e), 1 pt per component in (g),(i),(j). Otherwise no partial credit.)
- Topics: Solving equations, inequalities, and systems of equations, including those involving polynomials, rational functions, absolute values, logarithms, exponents, and trigonometric functions

10. (a)  $\frac{180t^\circ}{\pi}$ , (b)  $a$ , (c)  $a$ , (d)  $-a$ , (e)  $t$ , (f)  $\pi - t$

- 6 points: 1 point per part
- Topics: Trigonometric and Inverse Trigonometric Functions

11. (a) 10 ft, (b) 15 ft, (c) 15 ft,  
 (d) i.  $\frac{3}{5}$ , ii.  $\frac{4}{5}$ , iii.  $\frac{3}{4}$ , iv.  $\frac{4}{3}$ , v.  $\frac{5}{4}$ , vi.  $\frac{5}{3}$

- 6 points: 1 point per (a),(b),(c), 3 points (d)- 1/2 point per subpart
- Topics: Similar triangles, Pythagorean Theorem, Trigonometric functions