Given a homogeneous multivariable polynomial of degree two over the rational numbers, one is interested in counting the number of its integral solutions restricted to some box. In analytic number theory, one commonly estimates this number when the box gets bigger and bigger. Motivated by the work of others, my thesis studies a more refined estimate, which yields more information about this count, both algebraically and geometrically.

Motivated by the work of Heath-Brown and Getz, my thesis uses the circle method to prove a theorem on secondary terms in asymptotics for the number of zeros of quadratic forms in an odd number of variables $n$ whose coordinates are restricted in a smoothed box of size $B$. The result for the even degree case over arbitrary number fields was recently proved by Getz. In contrast to Getz's result, the corresponding secondary terms in the odd degree case involve the zero locus of a quadratic form in $n+1$ variables, not just a quadratic form in $n$ variables.