

# Tiebreaker Round

DMM 2022

## 1 Tiebreaker

**Problem 1:** The sequence  $\{x_n\}$  is defined by

$$x_{n+1} = \begin{cases} 2x_n - 1, & \text{if } \frac{1}{2} \leq x_n < 1 \\ 2x_n, & \text{if } 0 \leq x_n < \frac{1}{2} \end{cases}$$

where  $0 \leq x_0 < 1$  and  $x_7 = x_0$ . Find the number of sequences satisfying these conditions.

**Problem 2:** Let  $M = \{1, \dots, 2022\}$ . For any nonempty set  $X \subseteq M$ , let  $a_X$  be the sum of the maximum and the minimum number of  $X$ . Find the average value of  $a_X$  across all nonempty subsets  $X$  of  $M$ .