My thesis addresses two topics in dynamical systems: interaction of homeostasis and bifurcation and identifiability for a class of differential equations models. Homeostasis is a biological phenomenon in which a quantity doesn't change very much as an external quantity or parameter changes over a wide interval; body temperature is homeostatic with respect to environmental temperature, for example. Bifurcations are points in which the number of solutions to an equation changes when a parameter is varied. When homeostasis and bifurcation interact, it can lead to interesting behavior like systems which can switch between multiple homeostatic plateaus as in the picture. Identifiability asks whether the parameters of a model can be inferred from data, which is important when fitting or training a model.

One way to think about homeostasis mathematically is to study infinitesimal homeostasis points -- homeostasis at a point rather than on an interval. The marked points in the figure are homeostasis points. In my thesis, I studied the singularity that arises when these homeostasis points coincide with bifurcation points and classified all possible qualitative behaviors of systems with these singularities. I also characterize the structural identifiability of feedforward network models with linear kinetics. Interestingly, the set of reaction rates is identifiable, but the assignment of rates to reactions is not. Permutations of the reaction rates leads to the same measurements.